Lecture 24: Repeated eigenvalues; Nonhomogeneous systems (Sec. 7.8, 7.9)

MATH 308. Differential Equations

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November 29, 2022

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Repeated eigenvalues: general case

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If the 2×2 matrix A has repeated eigenvalues $\lambda = \lambda_1 = \lambda_2$ but is not $\begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix}$, then x_1 has the form $\boxed{x_1(t) = c_1 e^{\lambda t} + c_2 t e^{\lambda t}}$.

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Proof: the system $x' = Ax$ reduces to a second-order equation $x_1'' + p x_1' + q x_1 = 0$ with the same characteristic polynomial. This polynomial has roots $\lambda_1 = \lambda_2 = \lambda$, thus $x_1(t) = c_1 e^{\lambda t} + c_2 t e^{\lambda t}$.

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Finding x_2: If the first equation of the system contains x_2, we can express x_2 in terms of x_1.

Example

$$x_1' = 5x_1 + x_2, x_2' = -4x_1 + x_2$$

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Solve $x_1' = 2x_1 - x_2, x_2' = 2x_2$.
[Please write your answer in the chat]

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Just start from x_2 and use the second equation to express x_1.

Nonhomogeneous linear systems (Sec. 7.9)

- $x' = Ax$ — linear homogeneous system system with constant coefficients;
- $x' = A(t)x$ — linear homogeneous system;
- $x' = Ax + b(t)$ — linear nonhomogeneous system system with constant coefficients; $b(t)$ is a vector that may depend on t.
- $x' = A(t)x + b(t)$ — linear nonhomogeneous system.

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Example

Find the particular solution of the equation

$$x_1' = x_2 + 4, x_2' = x_1 + 1$$

that is a constant vector $(x_1(t), x_2(t)) = (a, b)$.
[Write your answer (a, b) in the chat]

Undetermined coefficients method: Guessing Table

If all components of $b(t)$ are [first column], then set all components of $x_p(t)$ to [second column] with different coefficients.

Guessing table for $x' = Ax + b(t)$

suppose $b(t) is$	then try for x_p
$ce^{\lambda t}$	$ke^{\lambda t}$
$c \sin at$ or $c \cos at$	$k \sin at + l \cos at$
Polynomial $p(t)$	Polynomial $q(t)$, same degree
$p(t)e^{\lambda t} \sin at$	$q_1(t)e^{\lambda t} \sin at + q_2(t)e^{\lambda t} \cos at$

Then substitute $x_p(t)$ into the equation and determine coefficients.

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- Answer: $x(t) = c_1 \begin{pmatrix} \cos t \\ -\sin t \end{pmatrix} + c_2 \begin{pmatrix} \sin t \\ \cos t \end{pmatrix} + \begin{pmatrix} 0.5e^t \\ -0.5e^t \end{pmatrix}$

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What if b contains terms from different rows of the Table?

Example

$x_1' = x_1 + x_2 + e^t$
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- Your guess for the particular solution will be the sum of the corresponding rows.

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Example

What is the correct guess for the particular solution
$x_1' = x_1 + 3x_2 + e^t$, $x_2' = 5x_2 + e^{2t}$?
[Write your answer in the chat]

Examples

Resonant case: when the guess does not work

- **If some of the summands in x_p also appear in x_c, you should add summands of the same kind multiplied by t.**

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$x_1' = x_2, x_2' = x_1 + 2e^t$

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- $$ae^t + be^t + bte^t = ce^t + dte^t,\ ce^t + de^t + dte^t = ae^t + bte^t + 2e^t$$
- $a + b = c, b = d, c + d = a + 2, d = b$

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$x_1' = x_2, x_2' = x_1 + 2e^t$

- New guess: $x_p(t) = (ae^t + bte^t, ce^t + dte^t)$.
- Substitute to find a, b, c, d:
- $$ae^t + be^t + bte^t = ce^t + dte^t,\ ce^t + de^t + dte^t = ae^t + bte^t + 2e^t$$
- $a + b = c, b = d, c + d = a + 2, d = b$
- The system may have several solutions, you need one of them.

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