MATH 308. Differential Equations

Final Exam Program

Nataliya Goncharuk, natasha_goncharuk@tamu.edu

Term Test 1 topics:

1. (Sec. 1.1-1.2) Determine if a given expression is a solution of the given differential equation.
2. (Sec. 1.1-1.2) Solve equations of the form $\frac{dy}{dx} = f(x)$.
3. (Sec. 1.1) Given a direction field or solution curves of a differential equation, make conclusions on the behavior of solutions with given initial conditions (intervals of increase/decrease, limits at infinity).
4. (Sec. 2.2) Solve separable equations $\frac{dy}{dx} = f(x)g(y)$. Remember to add constant solutions.
5. (Sec. 2.1) Solve linear first-order equations $y' + p(x)y = q(x)$.
6. (Sec. 2.4) Know and understand the statement of the Existence and Uniqueness theorem and be able to use it. If it does not apply, be able to explain why.
7. (Sec. 2.5) Given an autonomous equation $y' = f(y)$, construct the phase line, sketch the direction field and solution curves (make sure you know which is which). Make conclusions on the behavior of solutions with given initial conditions (increasing/decreasing, limits at infinity).

8. (Sec. 2.6) Solve exact equations. Determine if the given equation is exact.

9. (Sec. 3.1, 3.3, 3.4) Solve second-order linear homogeneous equations with constant coefficients using characteristic equations (in the cases of real, complex, and repeated roots).

10. (Sec. 3.2) Know the definition of the Wronskian of two solutions. Compute Wronskians, both directly and using the Abel's formula.

11. (Sec. 3.2) Know the definition of fundamental solutions and their relation to the general solution. Check if two given functions are fundamental solutions of the given linear second-order homogeneous equation.

12. (Sec. 3.5) Solve second-order linear nonhomogeneous equations using the Method of Undetermined Coefficients.

13. (Sec. 3.6) Solve second-order linear nonhomogeneous equations using Variation of Parameters.

14. (Sec. 3.7, 3.8) Study the motion of a given oscillator (damped or not damped, forced or not forced) using second-order linear equations. Make conclusions: is the motion periodic, what is its amplitude, will the system oscillate or not, what is its limit position (if it exists), will the motion of the system be bounded or unbounded.

Term Test 2 topics:

15. (Sec. 6.1) Compute Laplace transforms using a definition. Compute Laplace transforms and inverse Laplace transforms using a table of Laplace transforms.

16. (Sec. 6.3, 6.4) Solve first- and second-order differential equations with discontinuous right-hand side, or with right-hand side expressed as a piecewise function (using Laplace transforms).

17. (Sec. 6.5) Solve first- and second-order differential equations with delta-functions in the right-hand side (using Laplace transforms).

18. (Sec. 6.6) Compute convolutions of given functions.

19. (Sec. 6.6) For a given linear first- or second-order equation, express its solution using a convolution. Use this to solve a given equation.

20. If a second-order equation as above describes the motion of an oscillator, make conclusions on the motion of the oscillator: is the motion periodic, what is its amplitude, what is the limit position of the system (if it exists), will the motion of the system be bounded or unbounded.

21. (Sec. 5.2, 5.3) Solve ordinary differential equations using the Power Series method: find several first coefficients, or find the general formula for the coefficients of the power series expansion.

22. (Sec. 5.3) Use the Fuchs' theorem to determine the radius of convergence of the power series solution of the given second-order equation.

Later topics:

23. (Sec. 7.1-7.4) Solve linear systems of 2 equations with constant coefficients using the Eigenvalue method (real different eigenvalues).

24. (Sec. 7.6, 7.8) Solve linear systems of 2 equations with constant coefficients for complex or repeated eigenvalues.

25. (Sec 7.5, 9.1) Classify linear systems of 2 equations as saddle, node (sink/source), center, and spiral sink/source. Make conclusions on the behaviour of solutions of linear systems from the type of a system, or from its phase portrait.

26. (optional) Solve linear systems of 2 equations by reducing them to second-order equations.

27. (Sec.7.7) Find matrix exponentials e^{At} for 2×2 matrices A. Use the matrix exponential to solve the initial value problem $x' = Ax, x(0) = x_0$.

28. (Sec.7.9) Solve linear nonhomogeneous systems of 2 equations using the Undetermined Coefficients method.

29. (Sec.7.9) (optional) Solve linear nonhomogeneous systems of 2 equations using the Variation of Parameters method.

Meaning of "(Optional)" : the exam may contain problems that can be solved using several different menthods. A method marked as "optional" might be a shortcut, but you will always be able to solve the problem without it.