

Test 1

MATH 308 Sec 508

Texas A&M University, College Station

Nataliya Goncharuk

Oct 4, 2022, 11:10 pm — 12:20 pm (70 min)

Honor the Aggie Code: “An Aggie does not lie, cheat, or steal or tolerate those who do.”

Your Name:

Your UIN:

Task 1 (55 pt = 10+10+10+10+15 pt).

(a) Find the general solution of the following equation:

$$\frac{dy}{dx} + \frac{y}{x} = 1.$$

(b) Find a such that the equation

$$(x + 2y)\frac{dy}{dx} + (ay + \sin x) = 0$$

is exact. Solve the equation for this value of a . Implicit solutions are accepted.

(c) For the equation

$$\frac{dy}{dx} = \frac{(1 - 2y)^2}{x};$$

find the general solution and the solution with initial condition $y(1) = 0.5$.

Implicit solutions are NOT accepted.

(d) Find the general solution of the equation

$$y'' + 4y' + 3y = 0.$$

(e) Find the general solution of the equation

$$y'' - y = 2e^x$$

Task 2 (10+5 pt). The growth of the population of rabbits $y(x)$ (measured in thousands) is modeled using the equation

$$\frac{dy}{dx} = (y - 1)(y - 3)(y - 5).$$

- (a) Plot the phase line for this equation.
- (b) Determine the limit population $\lim_{x \rightarrow +\infty} y(x)$ if the initial population is $y(0) = 2.5$.

You are NOT required to solve the equation.

Task 3 (15 pt). The motion of the forced oscillator is governed by the equation

$$y'' + 9y = \sin x.$$

Find the general solution of this equation. Will the solution $y(x)$ stay bounded as $x \rightarrow +\infty$?

Task 4 (10+5 pt). (a) Prove that the Wronskian of any two solutions of the equation $x^2y'' - 2y = 0$ is constant.

(b) Find the general solution of the equation $x^2y'' - 2y = 0$.

Possible Hint: find two values of a such that $y(x) = x^a$ is a solution, then use the Superposition Principle.

End of the exam

Scrap paper. Will not be graded. If your solution continues here, write “solution continues on Scrap paper”.

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