Test 2

MATH 308 Sec 508
Texas A&M University, College Station

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Nov 10, 2022, 11:10 pm — 12:20 pm (70 min)

Honor the Aggie Code: "An Aggie does not lie, cheat, or steal or tolerate those who do."

Your Name:

Your UIN:

Task 1 (10+10+10 pt)**.** Compute the following Laplace transforms:

(a) $\mathcal{L}\{\delta(t-1)+(t-1)u_1(t)\}$;

(b) $\mathcal{L}\{(t^2e^t)'\}$;

(c) $\mathcal{L}\{(te^t)*e^t\}$.

Task 2 (10+5+5 pt)**.** (a) Compute the convolution $(te^t) * e^t$.

(b) Express the solution of the equation

$$y' - y = h(t), \quad y(0) = 0$$

as a convolution.

(c) Solve the equation $y' - y = te^t, \quad y(0) = 0$ (you may refer to a, b).

Sctratch paper.

Exam continues on the next page.

Task 3 (13+2 pt)**.** Someone hits an oscillator with a hammer. The equation for the motion of the oscillator is

$$y'' + 4y = \delta(t - \pi), \quad y(0) = 0, y'(0) = 1.$$

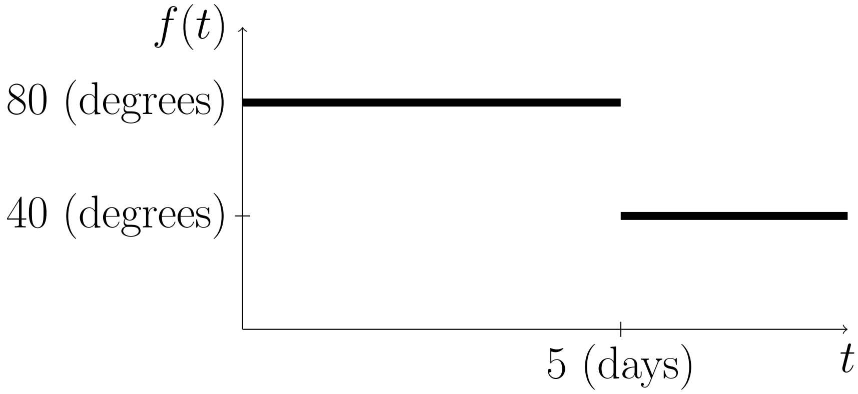
(a) Solve the equation using the Laplace transform.

(b) Find $y(2\pi)$.

Task 4 (8+7 pt)**.** The temperature in the unheated house $y(t)$ satisfies the following differential equation:

$$y'(t) + y(t) = f(t),$$

where the outside temperature $f(t)$ is given by the following graph.



The initial temperature in the house is $y(0) = 80$ (degrees).

(a) Show that

$$\mathcal{L}\{y\} = \frac{80}{s(s+1)} - \frac{40e^{-5s}}{s(s+1)} + \frac{80}{s+1}.$$

(b) Solve the differential equation. Find the limit temperature in the house $\lim_{t\to+\infty} y(t)$.

Solution of Task 4.

Exam continues on the next page.

Task 5 (20 pt)**.** Find the first 9 coefficients $a_0, a_1, \dots a_8$ of the power series solution $y(x) = \sum_{n=0}^{\infty} a_n x^n$ of the equation

$$y'' - x^2 y = 0, \quad y(0) = 1, y'(0) = 0.$$

End of the exam. The table of Laplace transforms is below.

$f(t)$	$F(s) = \mathcal{L}\{f\}(s)$
1	$\frac{1}{s}$, $s > 0$
e^{at}	$\frac{1}{s-a}$, $s > a$
t^n, $n = 1, 2, \dots$	$\frac{n!}{s^{n+1}}$, $s > 0$
$\sin(at)$	$\frac{a}{s^2+a^2}$, $s > 0$
$\cos(at)$	$\frac{s}{s^2+a^2}$, $s > 0$
$\sinh(at)$	$\frac{a}{s^2-a^2}$, $s > \|a\|$
$\cosh(at)$	$\frac{s}{s^2-a^2}$, $s > \|a\|$
$e^{at}\sin bt$	$\frac{b}{(s-a)^2+b^2}$, $s > a$
$e^{at}\cos bt$	$\frac{s-a}{(s-a)^2+b^2}$, $s > a$
$t^n e^{at}$, $n = 1, 2, \dots$	$\frac{n!}{(s-a)^{n+1}}$, $s > a$
$u_c(t) = \begin{cases} 0 & t < c \\ 1 & t \geq c \end{cases}$	$\dfrac{e^{-cs}}{s}$, $s > 0$ for $c \geq 0$
$u_c(t)f(t-c)$	$e^{-cs}F(s)$ for $c \geq 0$
$e^{ct}f(t)$	$F(s-c)$
$f(ct)$	$\frac{1}{c}F(\frac{s}{c})$ for $c > 0$

$f(t)$	$F(s) = \mathcal{L}\{f\}(s)$	
$(f * g)(t) = \int_0^t f(t-\tau)g(\tau)d\tau$	$F(s)G(s)$	
$\delta(t-c)$	e^{-cs}	for $c \geq 0$
$f^{(n)}(t)$	$s^n F(s) - s^{n-1}f(0) - \cdots - f^{n-1}(0)$	
$(-t)^n f(t)$	$F^{(n)}(s)$	

Do not write your solutions here — this page will not be graded.