

MATH 308. Differential Equations

Quiz 1

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Task 1. For the differential equation $y' = -y^2$, answer the following questions.

- (a) Solve the equation.
- (b) Sketch the direction field.
- (c) For the solution of this equation with initial condition $y(0) = 1$, find its limit as $x \rightarrow +\infty$.

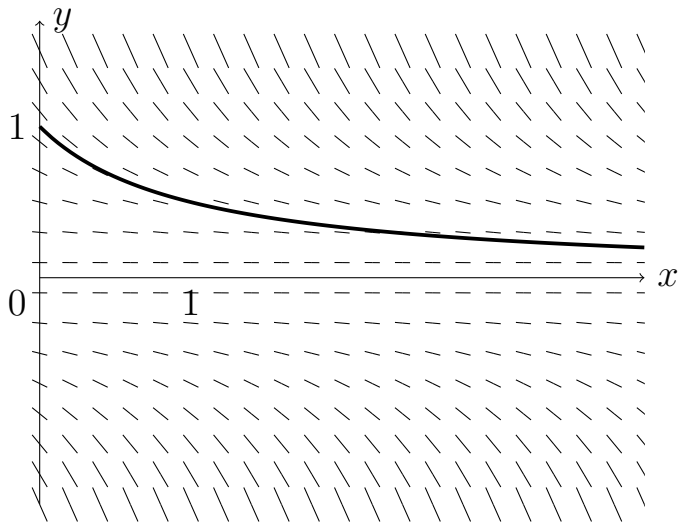
(a) Using Separation of Variables, we get

$$\int -\frac{dy}{y^2} = \int dx,$$

thus $1/y = x + c$ and $y(x) = \frac{1}{x+c}$. We have lost the constant solution $y(x) = 0$ when we divided by y ; this solution should be added.

Answer: $y(x) = \frac{1}{x+c}$ for any c and $y(x) = 0$.

(b) Direction field appears below, with the solution curve corresponding to (c).



(c) As we can see from the direction field, the solution with initial condition $y(0) = 1$ decreases, and its graph is asymptotic to $y = 0$. Thus $\lim_{x \rightarrow +\infty} y(x) = 0$. Alternatively, we may notice that the solution with initial condition $y(0) = 1$ is given by $y(x) = \frac{1}{x+1}$ and $\lim_{x \rightarrow +\infty} \frac{1}{x+1} = 0$.
