MATH 308. Differential Equations

Quiz 2 Solutions

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Task 1 (5 pt): Solve the equation $y' + 2y/x = x$.

Solution: this is a linear equation. The integrating multiplier is $\mu(x) = \exp \int 2/x dx = \exp(2\ln|x|) = x^2$. We get

$$x^2 y' + 2xy = x^3 \Rightarrow (x^2 y)' = x^3 \Rightarrow x^2 y = \frac{x^4}{4} + c$$

thus $y(x) = \frac{x^4/4+c}{x^2} = x^2/4 + c/x^2$.

Task 2 (5 pt): Solve the equation $y'(y^3 + x) + (y + 1/x) = 0$. Implicit solutions are accepted.

Solution: this equation is exact since $\frac{\partial}{\partial x}(y^3 + x) = 1 = \frac{\partial}{\partial y}(y + 1/x)$.

Find the function $\psi(x, y)$ such that

$$\psi'_x = y + 1/x, \qquad \psi'_y = y^3 + x.$$

The first equation implies

$$\psi(x, y) = \int (y + 1/x) dx + h(y) = xy + \ln|x| + h(y).$$

Now the second equation implies

$$h'(y) + (xy + \ln|x|)'_y = y^3 + x,$$

i.e. $h'(y) = y^3$ and $h(y) = y^4/4$.

We conclude that $\psi(x, y) = xy + \ln|x| + y^4/4$ has required partial derivatives. Thus our equation is equivalent to $\frac{d}{dx}\psi(x, y) = 0$ and its solution is given implicitly by $\psi(x, y) = c$.

Final answer: $xy + \ln|x| + y^4/4 = c$