MATH 308. Differential Equations

Quiz 3 Solutions

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Task 1 [5 pts]. Find the Laplace transform of the function $f(t)$ that equals 1 for $t < 5$ and 0 for $t > 5$.

Solution: This function equals $1 - u_5(t)$, thus its Laplace transform is $\mathcal{L}\{1 - u_5(t)\} = \frac{1}{s} - \frac{e^{-5s}}{s}$.

Alternatively, we can compute the integral directly: $\mathcal{L}\{f\} = \int_0^\infty e^{-st} f(t)dt = \int_0^5 e^{-st}dt = \frac{e^{-st}}{-s}\Big|_0^5 = \frac{1}{s} - \frac{e^{-5s}}{s}$.

Task 2a [3 pts]. Show that the inverse Laplace transform of $\frac{1}{s-s^2}$ equals $1 - e^t$.

Task 2b [2 pts]. Find the inverse Laplace transform of $\frac{e^{-s}}{s-s^2}$ using Task 2a.

Solution.

2a: Indeed, $\mathcal{L}\{1-e^t\} = \frac{1}{s} - \frac{1}{s-1} = \frac{-1}{s(s-1)} = \frac{1}{s-s^2}$, thus $\mathcal{L}^{-1}\{\frac{1}{s-s^2}\} = 1 - e^t$. We can also do this computation "right to left", using partial fractions to represent $\frac{1}{s-s^2}$ as $\frac{1}{s} - \frac{1}{s-1}$.

2b: Using the "shift formula" $\mathcal{L}\{u_c(t)f(t-c)\} = e^{-cs}\mathcal{L}\{f\}$ for $c = 1$ and $f(t) = 1 - e^t$ from 2a, we get

$$\mathcal{L}\{u_1(t)(1 - e^{t-1})\} = e^{-cs}\mathcal{L}\{f\} = \frac{e^{-s}}{s - s^2},$$

thus the answer is $u_1(t)(1 - e^{t-1})$.