MATH 308. Differential Equations

Quiz 5 solutions

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Some matrix A has two eigenvalues, -2 and 3. The corresponding eigenvectors are $(1, 1)$ and $(1, 0)$, respectively.

(a) Find the general solution of the system $x' = Ax$.

(b) Find the solution with initial condition $x_1(0) = 5, x_2(0) = 5$. Write your answer in the form $x_1(t) = ..., x_2(t) = ...$

(c) Is it true that all solutions of the system $x' = Ax$ tend to zero (i.e. both $x_1(t)$ and $x_2(t)$ tend to zero) as $t- > +\infty$?

(a) Due to the formula of the Eigenvalue method,

$$x(t) = c_1 e^{-2t} \begin{pmatrix} 1 \\ 1 \end{pmatrix} + c_2 e^{3t} \begin{pmatrix} 1 \\ 0 \end{pmatrix}.$$

(b) Substituting $t = 0$, we get $c_1 + c_2 = 5, c_1 = 5$, thus $c_1 = 5$ and $c_2 = 0$. The solution is thus $x(t) = 5e^{-2t} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$, i.e.

$$x_1(t) = 5e^{-2t}, x_2(t) = 5e^{-2t}$$

(c) The general solution of the system was found in (a):

$$x(t) = c_1 e^{-2t} \begin{pmatrix} 1 \\ 1 \end{pmatrix} + c_2 e^{3t} \begin{pmatrix} 1 \\ 0 \end{pmatrix}.$$

This does NOT always tend to zero: while the first summand tends to zero (since $e^{-2t} \to 0$), the second summand for $c_2 \neq 0$ will not tend to zero since the exponentials e^{3t} tend to infinity.

We can make the same conclusion if we note that the system is a saddle (one eigenvalue is positive and one is negative), and hyperbola-like curves of a saddle do not tend to zero.

Remark: Note that in (b), we have found ONE solution of the system $x' = Ax$*, and THIS ONE solution tends to zero. However, (c) was asking about ALL solutions of the system, not just the one from (b).*