MATH 308. Differential Equations

Homework 4

Nataliya Goncharuk, natasha_goncharuk@tamu.edu

Deadline: Feb 16, 10:00 pm

Task 1 (3 pt)**.** Solve the following equations:

a) $y'' + 4y = 0$;

b) $y'' + 4y' = 0$;

c) $y'' + 4y' + 4y = 0$;

d) $y''' - y'' + 4y' - 4y = 0$;

e) $y^{(4)} - 6y^{(2)} + 9y = 0$;

f) $y^{(4)} - y = 0$

Your final answer should not include complex exponentials.
You may check roots of your characteristic polynomials using a computer algebra system.

Task 2. (3 pt) The damped oscillator is governed by the equation $x'' = -4x - cx'$ where c is a damping coefficient.

a) (overdamped case) For $c = 5$, find the general solution and the solution with initial condition $x(0) = -2$, $x'(0) = 14$. Show that x changes sign only once for $t > 0$.

b) (critically damped case) For $c = 4$, find the general solution and the solution with initial condition $x(0) = -2$, $x'(0) = 14$. Show that x changes sign only once for $t > 0$.

c) (underdamped case) For $c = 2$, find the general solution and the solution with initial condition $x(0) = -2$, $x'(0) = 14$. Show that x changes sign infinitely many times for $t > 0$.

Task 3. (2+2+2: item (c) gives 2 bonus pts) Consider the linear homogeneous equation with non-constant coefficients of the form

$$x^2y'' + bxy' + cy = 0 \tag{1}$$

where b, c are real numbers.

a) Verify that $y = x^\alpha$ is a solution of (1) if and only if α is a root of the characteristic polynomial

$$\alpha(\alpha - 1) + b\alpha + c = 0 \tag{2}$$

Use this to find two independent solutions of the equation

$$x^2y'' + 2xy' - 6y = 0$$

and write out its general solution.

b) For complex α, define $x^\alpha := e^{\alpha \ln x}$. It turns out that the real and the imaginary part of x^α will be solutions of the equation (1) if α satisfies (2). Use this to solve the equation

$$x^2y'' + xy' + 4y = 0.$$

Your final answer should not include complex exponentials.

c) Now solve the equation

$$x^2y'' + 5xy' + 4y = 0.$$

for which the characteristic equation has a repeated root. To do this, find solutions in the form $y(x) = g(x)x^\alpha$ where α is this repeated root: substitute $y(x)$ into the equation and find g. This method is called the reduction of order.