MATH 308. Differential Equations

Homework 5

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Deadline: Mar 2, 10:00 pm

Task 1. (2+1 bonus pt) (a) Guess one solution of the equation $y'' + xy' - (x+1)y = 0$ (hint: try exponentials) and find all solutions using reduction of order. Your final answer may contain integrals.

(b)(*) Find the solution with initial conditions $y(0) = 1, y'(0) = 2$. Your final answer may contain integrals.

Task 2 (1+1+1+1 pt)**.** Solve the following nonhomogeneous ODEs.

(a) $y'' = y + e^x \cos(2x)$;

(b) $y'' - 4y' + 3y = 9x^2 + 9$;

(c) $y'' + 2y' + y = 8 \cosh x$;

Comment: cosh is a hyperbolic cosine,

$$\cosh x = \frac{e^x + e^{-x}}{2}$$

(d) $(\frac{d}{dx} - 1)^5 y = e^x \sin x$. Recall that $(\frac{d}{dx} - 1)^5$ is a differential operator $(\frac{d}{dx} - 1)$ applied five times.

Hint: denote $y = e^x g(x)$ and use formulas from Lec 10

Task 3 (2+1+1 pt)**.** The periodic outer force $F = \sin \omega t$ is applied to a damped oscillator. The motion off the oscillator is described by the equation

$$y'' = -3y - cy' + \sin \omega t,$$

$c > 0$.

(a) Find a periodic solution of this equation in the form $y(t) = a \cos \omega t + b \sin \omega t$ (the answer will depend on c, ω). Prove that the amplitude of this solution $A = \sqrt{a^2 + b^2}$ is equal to $((3 - \omega^2)^2 + c^2\omega^2)^{-0.5}$.

(b) For $c = 2$, write out the general solution of this equation and show that all solutions tend to the periodic solution as $t \to +\infty$.

(c) For any c (you may assume that c is small) find $\omega = \omega_{max} > 0$ such that the amplitude of this periodic solution reaches its maximum at ω. You will find the "resonant" frequency of the outer force for the damped oscillator.

Comment: you will see that for larger c, the amplitude monotonically decreases as ω increases, and thus there is no maximum point $\omega_{max} > 0$.

Hint: you may wish to denote $\nu = \omega^2$ when searching for the maximum, but remember that $\nu \geq 0$.