Test 2

MATH 308 Sec 200
Texas A&M University, College Station

Nataliya Goncharuk

April 4, 2023, 11:10 pm — 12:25 pm (75 min)

Honor the Aggie Code: "An Aggie does not lie, cheat, or steal or tolerate those who do."

Your Name:

Your UIN:

Task 1 (13+2 pt)**.** The motion of the forced oscillator is described by the equation

$$y'' + y = 3\sin 2t + 1.$$

(a) Find the solution of this equation with initial conditions $y(0) = 0, y'(0) = 0$.

(b) Show that the motion is bounded: $|y(t)| \leq 5$ for all t.

Task 2 (10+10 pt)**.** (a) Solve the equation $y'' - 4y' + 3y = e^t$.

(b) Solve the equation $y'' - 4y' + 3y = e^{at}$ for all values of parameter a such that $a \neq 1, 3$.

Task 3. (5+10 pt) The matrix exponential of the matrix A equals $e^{At} = \begin{pmatrix} e^t & e^t - 1 \\ 0 & 1 \end{pmatrix}$.

(a) Solve the linear system $x' = Ax$ with initial condition $x(0) = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$.

(b) Solve the system $x' = Ax + \begin{pmatrix} 1 \\ e^t \end{pmatrix}$ using variation of parameters.

You are not required to determine the matrix A.

Task 4 (10 pt)**.** Solve the system $x' = Ax$ and find the matrix exponential e^{At} where $A = \begin{pmatrix} 0 & 2 \\ 2 & 0 \end{pmatrix}$.

Task 5 (15 pt)**.** Solve the system

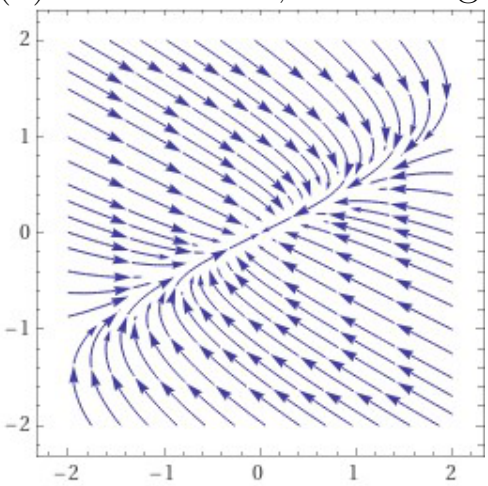
$$x_1' = -x_1 + x_2$$
$$x_2' = -x_2 + 9e^{2t}$$

Task 6 (15+5+5 pt)**.** Consider the system

$$x_1' = ax_1 + 4x_2,$$
$$x_2' = x_1 + ax_2$$

Its phase portrait for $a = -3$ is shown below.

(a) For $a = -3$, find the general solution of the system.



(b) For $a = -3$, find the solution with initial condition $x_1(0) = 0, x_2(0) = 2$. Using either your formula for $x(t)$ or a picture, show that for this solution, $x_1(t) > 0$ for all $t > 0$.

(c) For all a (except the degenerate case $a = \pm 2$), determine the type of the equilibrium for the linear system (saddle, node sink/source, spiral sink/source, center). For which a the equilibrium is stable?

End of the exam

Scratch paper

Scratch paper