MATH 308. Differential Equations

Final Exam Program

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1. (Sec. 1.1-1.2) Determine if a given expression is a solution of the given differential equation. For a given expression with parameters, determine values of parameters such that the expression solves the equation. Find constant solutions of a given equation.

2. **Direction fields** (Sec. 1.1) Given a direction field or solution curves of a differential equation, make conclusions on the behavior of solutions with given initial conditions (intervals of increase/decrease, limits at infinity).

3. **Separable** (Sec. 2.2) Solve separable equations $\frac{dy}{dx} = f(x)g(y)$. Remember to add constant solutions.

4. **Linear** (Sec. 2.1) Solve linear first-order equations $y' + p(x)y = q(x)$.

5. **Existence and Uniqueness** (Sec. 2.4) Know and understand the statement of the Existence and Uniqueness theorem and be able to use it. If it does not apply, be able to explain why.

6. **Phase line** (Sec. 2.5) Given an autonomous equation $y' = f(y)$, construct the phase line, sketch the direction field and solution curves (make sure you know which is which). Make conclusions on the behavior of solutions with given initial conditions (increasing/decreasing solutions, limits at infinity).

7. **Exact** (Sec. 2.6) Solve exact equations. Determine if the given equation is exact. Guess an integrating multiplier using a hint.

8. **Change of Variables** (see notes for Lec 6) Given a change of variable, use it to solve the given equation.

9. **Higher-order** (Sec. 3.1, 3.3, 3.4, 4.2) Solve higher-order linear homogeneous equations with constant coefficients using characteristic equations (in the cases of real, complex, and repeated roots).

10. **Undetermined Coefficients** (Sec. 3.5) Solve second-order linear nonhomogeneous equations using the Method of Undetermined Coefficients.

11. **Variation of Parameters** (Sec. 3.6) Solve second-order linear nonhomogeneous equations using Variation of Parameters (directly as in Sec. 3.6 or by reducing them to systems and using Sec. 7.9).

12. **Wronskians, Abel's Formula, Fundamental solutions, Reduction of Order** (Sec. 3.2, 3.4) For second-order linear homogeneous equations, know the definition of fundamental solutions and their relation to the general solution. Check if given functions are fundamental solutions of the given linear homogeneous second-order equation. Know Abel's formula. Find fundamental systems of solutions using hints.

13. **Oscillators** (Sec. 3.7, 3.8) Study the motion of a given oscillator (damped or not damped, forced or not forced) using second-order linear equations. Make conclusions: is the motion periodic, will the system oscillate or not, what is its limit position (if it exists), will the motion of the system be bounded or unbounded.

14. **Linear systems** (Sec. 7.1-7.4) Solve linear systems of 2 equations with constant coefficients using the Eigenvalue method and/or the elimination method [Real, Complex, Repeated roots].

15. **Phase portraits in 2D** (Sec 7.5, 9.1) Classify linear systems of 2 equations as saddle, node (sink/source), center, and spiral sink/source.

Sketch phase portraits, determine stability, make conclusions on the behaviour of solutions.

16. **Fundamental matrices** (Sec.7.7) Know the definition of the fundamental matrix. Find fundamental matrices for systems $x' = Ax$ with 2×2 matrices A. Use fundamental matrices to write out the general solution of the equation.

17. **Matrix Exponentials** (Sec.7.7) Find matrix exponentials e^{At} for 2×2 matrices A. Use the matrix exponential to solve the initial value problem $x' = Ax, x(0) = x_0$.

18. **Undetermined coefficients - 2D** (Sec.7.9) Solve linear nonhomogeneous systems of 2 equations using the Undetermined Coefficients method.

19. **Variation of Parameters -2D** (Sec.7.9) Solve linear nonhomogeneous systems of 2 equations using the Variation of Parameters method.

20. **Nonlinear Systems** (Sec. 9.2-9.4) Find equilibriums(=critical points = singular points) of nonlinear systems, determine their types and stability. Use the phase portrait to determine the limit behaviour of a solution with given initial condition.

21. **Laplace transforms** (Sec. 6.1 -6.4) Find Laplace transforms of given functions using the definition and the Table of Laplace transforms. Use Laplace transforms to solve linear second-order nonhomogeneous equations with discontinuous right-hand side and with delta-functions (to model impacts).

22. **Convolution** (Sec. 6.5) Know the definition of convolution. Represent solutions of linear second-order equations as convolution integrals.

23. **Power series method** (Sec. 5.1-5.3) Solve differential equations using the Power Series method.