MATH 617. Complex Analysis

Homework 9

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Deadline: Monday Nov 6, 10 pm

Task 1. (2+2 pt) (a) Use computer-generated plots to determine the number of zeros of the function $f(z) = z^4 + iz + 0.5$ in the disc $|z| < 1$. Check that there are no repeated zeros in $|z| < 1$.

(b) Use Rouche's theorem to determine the number of zeros of the function $f(z) = z^6 + z^3 + 3z + 1$ in the ring $0.9 < |z| < 3$ (counting with multiplicity).

Task 2. (2 pt) Find the Laurent series expansion for the function $\dfrac{1}{z^2(z-1)}$ in $|z| > 1$.

Task 3. (1+1+1+1 pt) Suppose that f is analytic in the domain $|z| > R$. We will say that f has a removable singularity/a pole of order k/ an essential singularity at infinity if $f(1/z)$ has a removable singularity/ a pole of order k/ an essential singularity at $z = 0$.

(a) Prove that a function f has a removable singularity at infinity if and only if its Laurent series expansion in $|z| > R$ only contains terms with non-positive indices, $f(z) = \sum_{n=-\infty}^{0} a_n z^n$.

(b) Prove that if a function f has a pole of order k at infinity, then it has the form $f(z) = p(z) + h(z)$ where p is a polynomial of degree k and h has a removable singularity at infinity.

(c) Prove that if an entire function has a removable singularity at infinity, then it is constant.

(d) Prove that an entire function has a pole at infinity of order k if and only if it is a polynomial of degree k.