MATH 618. Complex Analysis

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Task 1. (a) Give a definition of an analytic (1-dimensional) manifold.

(b) Let $\pi\colon \mathbb{C} \to \mathbb{C}/\mathbb{Z}$ be the natural projection. If $\phi_1 = \pi$ on $D_1 = \{|z| < 0.5\}$ and $\phi_2 = \pi$ on $D_2 = \{|z-1| < 0.5\}$ are used as coordinate patches on the manifold $\mathbb{C}/\mathbb{Z}$, then what is the transition map between these charts and what is its domain?

Task 2. (a) Suppose that $\sin z - z$ never takes a value a. Show that it never takes the value $a + 2\pi$.

(b) Derive that the equation $\sin z - z = a$ has a solution for all complex a.

Task 3. Consider the multivalued function $\log(1+\sqrt{z})$. Let

$$U = \mathbb{C} \setminus (\{x \in \mathbb{R}, x < 0\} \cup \{x \in \mathbb{R}, x > 1\})$$

be a plane with two slits.

(a) Sketch images of U under all branches of this function in U. Sketch images of the curves $\gamma_1 = \{|z| = 0.5\}$ and $\gamma_2 = \{|z-1| = 0.5\}$.

(b) Consider the branch f of this function such that $f(0.5)$ equals the (real) logarithm of one plus (positive) square root of 0.5. Find the value at 0.5 of the analytic extension of f along the path $\gamma_1\gamma_2$. Here we first go along γ_1 and then along γ_2; both curves are oriented counterclockwise.

Task 4. The function f is analytic in a half-disc $\{|z| < 1, \operatorname{Im} z > 0\}$. It extends continuously to all points of the boundary except 1. Its values on the boundary are real. Show that it extends analytically to $\mathbb{C} \setminus \{1\}$.