$$(1)^n \begin{cases} u_t = \Delta u := \sum_{i=1}^{n} \frac{\partial^2}{\partial x_i^2} u \\ u(x,0) = f(x) := f(x_1, x_2, \dots, x_n) \end{cases}$$

$$G(x_1, x_2, \dots, x_n, t) = C_n \, e^{-\frac{|x|^2}{4t}}, \quad \text{recall } c_1 = \frac{1}{(4\pi t)^{1/2}}$$

$$\int_{\mathbb{R}^n} G(x,t)\,dx = C_n \cdot \int_{\mathbb{R}^n} e^{-\frac{x_1^2 + \dots + x_n^2}{4t}} dx_1 \dots dx_n$$

$$= C_n \cdot \prod_{i=1}^{n} \int_{\mathbb{R}} e^{-\frac{x_i^2}{4t}} dx_i$$

$$= C_n \left(\int_{\mathbb{R}} e^{-\frac{x^2}{4t}} dx \right)^n = 1$$

Recall $c_1 \cdot \int_{\mathbb{R}} e^{-\frac{x^2}{4t}} dx = 1$, then $C_n = (c_1)^n$

and we derive:

$$G(x,t) = \frac{1}{(4\pi t)^{\frac{n}{2}}} e^{-\frac{|x|^2}{4t}}$$

with a "candidate" for solution of $u_t = \Delta u$:

$$u(x,t) = \int_{\mathbb{R}^n} f(\bar{x})\, G(x - \bar{x}, t)\, d\bar{x}$$