Problem 2. Solve the PDE

$$\begin{aligned}
&\partial_{tt}u = \partial_{xx}u, && && 0<x<2,\ t>0,\\
&u(0,t)=0, && u(2,t)=0, && t>0,\\
&u(x,0)=0, && \partial_t u(x,0) = 2\pi\sin(2\pi x), && 0<x<2.
\end{aligned}$$

$f(x) = 0 \qquad g(x) = 2\pi \sin(2\pi x)$

Odd periodic extensions = $\bar{f}(x)$ and $\bar{g}(x)$

Rules: $\begin{cases} ① \ \bar{f}(-x) = -f(x) \text{ for } 0<x<2 \\ ② \ \bar{f}(x+4) = \bar{f}(x), \text{ i.e., 4-periodic function } \bar{f}(x) \end{cases}$

$\{$... Same for $g(x) \Rightarrow ① \ \bar{g}(-x) = -g(x)$ (odd)

② and 4 periodic (outside $[-2,2]$)

Note: $\sin 2\pi x$ is odd and 4 periodic!

$\Rightarrow \bar{g}(x) \equiv g(x)$.

Then by D'Alambert's formulas

$$u(x,t) = \frac{1}{2}(0+0) + \frac{1}{2}\int_{x-t}^{x+t} 2\pi \sin(2\pi y)\,dy$$

$$u(x,t) = -\cos(2\pi y)\Big|_{x-t}^{x+t} = \underline{\cos 2\pi(x-t) - \cos 2\pi(x+t)}$$