

**Problem 4.** Consider the conservation equation

$$\partial_t \rho + \partial_x (\rho^2 + \rho) = 0, \quad x \in (-\infty, \infty), \quad t > 0$$

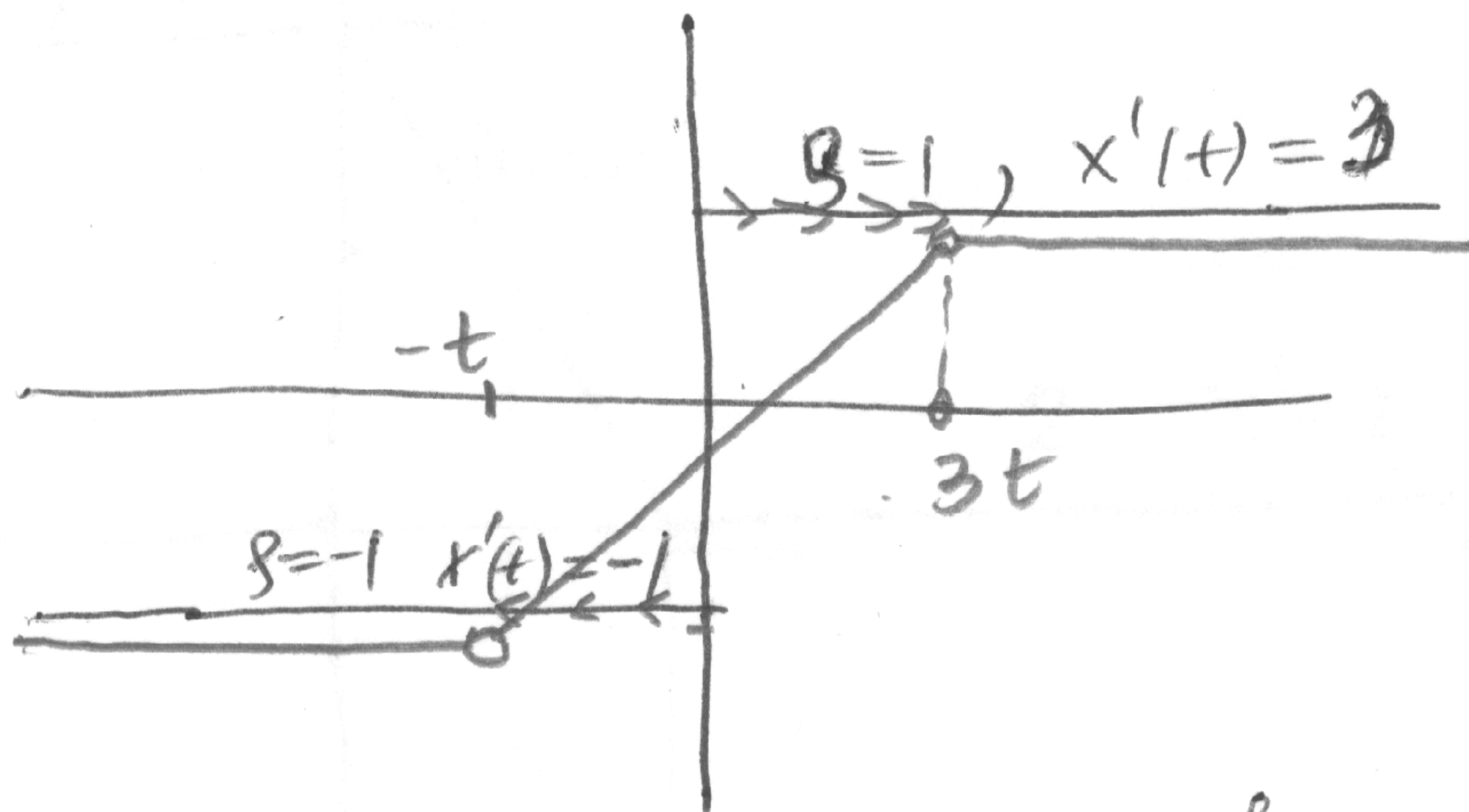
with the initial condition

$$\rho(x, 0) = -1, \quad \text{if } x < 0,$$

$$\rho(x, 0) = 1, \quad \text{if } x > 0.$$

$$\rho_t + (2\rho + 1)\rho_x = 0$$

Solve this problem using the method of characteristics. Do we have a shock or an expansion wave here?



$$u(x, t) = \begin{cases} -1 & \text{if } x < -t \\ \text{linear} = \text{find it} & \text{if } -t < x < 3t \\ 1 & \text{if } x > 3t \end{cases}$$