Solution Key for Assignment 6, 412, Applied PDE

12.2.3

$$
\begin{aligned}
x - 4t &= x_0 \\
w(x,t) &= P(x-4t) \\
w(0,t) &= \sin(3t) \\
w(x,t) &= \sin(-\frac{3}{4}(x-4t)) \\
&= \sin(3t - \frac{3}{4}x)
\end{aligned}
$$

12.2.4

$$
\begin{aligned}
&x = ct + x_0 \\
&w(x,t) = w(x_0,0) = f(x_0) = f(x-ct), x > ct \\
&x = c(t-t_0) \Rightarrow t_0 = t - \frac{x}{c} \\
&w(x,t) = w(0,t_0) = h(t_0) = h(t-\frac{x}{c}), x < ct
\end{aligned}
$$

12.2.7 (a)

$$
\begin{aligned}
\frac{dx}{dt} = 2u, \frac{du}{dt} = 0 \\
x = 2u(x_0,0)t + x_0
\end{aligned}
$$

12.2.7 (b)

1.

$$
\begin{aligned}
&x_0 < 0 \Rightarrow x - 2f(x_0)t < 0, \\
&u(x,t) = 1, x < 2t;
\end{aligned}
$$

2.

$$
\begin{aligned}
&0 < x_0 < L \Rightarrow 0 < \frac{x-2t}{1+\frac{2t}{L}} < L \Rightarrow 2t < x < L + 4t \\
&f(x_0) = 1 + \frac{x_0}{L} \\
&x = 2(1+\frac{x_0}{L})t + x_0 \Rightarrow x_0 = \frac{x-2t}{1+\frac{2t}{L}} \\
&u = \frac{L+x}{1+\frac{2t}{L}}
\end{aligned}
$$

3.

$$
\begin{aligned}
&x_0 > L \Rightarrow x - 2f(x_0)t > L \Rightarrow x > L + 4t \\
&f(x_0) = 2 \\
&x = 4t + x_0 \\
&u = 2.
\end{aligned}
$$

12.4.1

$$u(x,t) = F(x-ct) + G(x+ct)$$
1.
$$x > ct, G(x+ct) = 0, F(x-ct) = 0$$
$$\Rightarrow u(x,t) = 0;$$
2.
$$x < ct, G(x+ct) = 0$$
$$u(x,t) = u(0,t_0) = h(t_0) = h(t - \frac{x}{c})$$

12.4.2

$$u(x,t) = F(x-ct) + G(x+ct)$$
$$f(x) = \cos x, g(x) = 0$$
1.
$$x < 0, F(x) = G(x) = \frac{1}{2}\cos x$$
$$u(0,t) = e^{-t} = F(-ct) + G(ct), t > 0$$
2.
$$x < -ct, F(x) = G(x) = \frac{1}{2}\cos x$$
$$u(x,t) = \frac{1}{2}\cos(x-ct) + \frac{1}{2}\cos(x+ct)$$
$$= \cos x \cos ct$$
3.
$$0 > x > -ct, G(x+ct) > 0$$
$$G(x+ct) = e^{-\frac{(x+ct)}{c}} - F(-x-ct)$$
$$u(x,t) = e^{-\frac{(x+ct)}{c}} - F(-x-ct) + F(x-ct)$$
$$= e^{-\frac{x}{c}-t} + \frac{1}{2}\cos(x-ct) - \frac{1}{2}\cos(x+ct)$$
$$= e^{-\frac{x}{c}-t} + \sin x \sin ct$$