

Math 166 - Week in Review #3 - Exam 1 Review

NOTE: For reviews of the other sections on Exam 1, refer to the first page of WIR #1 and #2.

Section 3.1 - Graphing Systems of Linear Inequalities

- NOTE: The following procedure is for shading the **TRUE** region. (Some instructors illustrate shading the “false” region in their classes.)

Procedure for Graphing Linear Inequalities

1. Draw the graph of the equation obtained for the given inequality by replacing the inequality sign with an equal sign.
 - i) If the inequality is a STRICT inequality (either $<$ or $>$), use a dashed or dotted line when graphing the equation.
 - ii) If the inequality is inclusive (either \leq or \geq), use a solid line to indicate that the line itself is part of the solution set.
 2. Pick a test point lying in one of the half planes determined by the line sketched in Step 1. Substitute these values for x and y into the ORIGINAL inequality. Use the origin whenever possible.
 3. i) If plugging in the test point makes the inequality TRUE, shade the side of the line that INCLUDES the test point.
 - ii) If plugging in the test point makes the inequality FALSE, shade the half of the plane that DOES NOT INCLUDE the test point.
- The solution set of a system of linear inequalities is bounded if it can be enclosed by a circle. Otherwise, it is unbounded.

Section 3.2 - Linear Programming Problems

- A linear programming problem consists of a linear objective function to be maximized or minimized subject to certain constraints in the form of linear equations or inequalities.
- When asked to set up or formulate a linear programming word problem, you are expected to
 1. Define the variables.
 2. State the objective (goal) *and* the objective function.
 3. State the constraints.
- For many linear programming word problems, setting up a table to summarize the information given in the problem is very helpful. However, there are times when not all of the constraints of the problem fit nicely in the table, so be sure to re-read the problem after making the table to see if there is any other information you have missed and that should be included as an inequality in the constraints.

Section 3.3 - Graphical Solution of Linear Programming Problems

- If a linear programming problem has a solution, then it will occur at a corner point (a.k.a vertex) of the feasible set S that is determined by the constraints of the problem.
- If S is bounded, then the objective function has both a maximum and a minimum value on S .
- If S is unbounded and the coefficients of the variables in the objective function are both nonnegative, the the objective function has a minimum value on S provided that $x \geq 0$ and $y \geq 0$ are two of the constraints.
- If S is the empty set, then the linear programming problem has no solution (i.e. the objective function has neither a maximum nor a minimum value).

- If the objective function is optimized at two adjacent vertices of S , then it is optimized at every point on the line segment joining these vertices. In this case, there are infinitely many solutions to the problem.
- **The Method of Corners for a BOUNDED Feasible Set**
 - Step 1: Graph the feasible set.
 - Step 2: Find the coordinates of all corner points of the feasible set.
 - Step 3: Evaluate the objective function at each corner point.
 - Step 4: Find the vertex which gives the maximum (minimum) value of the objective function.
 - If there is only one such vertex, then this vertex is the unique solution to the problem.
 - If the objective function is maximized (minimized) at two adjacent corner points of S , then there are infinitely many solutions given by the points on the line segment joining these two vertices.
- NOTE: For an **UNBOUNDED** solution set, the procedure for the Method of Corners is the same, but depending on the coefficients of your objective function, a maximum value or a minimum value of the objective function may not be possible.

1. True/False

- | | | |
|------|-------|---|
| TRUE | FALSE | a) $I_n A = A I_n = A$ for all matrices A . |
| TRUE | FALSE | b) A nonsingular matrix has no inverse. |
| TRUE | FALSE | c) To be able to compute the matrix product AB , the number of columns of a must equal the number of rows of B . |
| TRUE | FALSE | d) If B is a 2×2 matrix, then $B + I_2 = B$. |
| TRUE | FALSE | e) When solving the matrix equation $AX = B$ by computing $A^{-1}B$ in the calculator, a message of "ERR: SINGULAR MAT" implies that the system of equations has no solution. |
| TRUE | FALSE | f) Every square matrix has an inverse. |
| TRUE | FALSE | g) If the parametric solution to a system of equations is $(3t + 2, -t - 3, t)$, then $(-7, 0, -3)$ is a particular solution. |

2. Find the value of k so that the following system has no solution:
- $$\begin{array}{rcl} 5x + 2y & = & -7 \\ -3x + ky & = & 8 \end{array}$$

3. For each of the following, if the matrix is in row reduced form, interpret its meaning as a solution to a system of equations. If the matrix is not in row reduced form, explain why.

$$(a) \left[\begin{array}{ccc|c} 1 & 0 & 0 & 4 \\ 0 & 1 & 3 & -5 \\ 0 & 0 & 1 & 0 \end{array} \right]$$

$$(e) \left[\begin{array}{ccc|c} 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 4 \\ 1 & 0 & 0 & 1 \end{array} \right]$$

$$(b) \left[\begin{array}{ccc|c} 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 8 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

$$(f) \left[\begin{array}{ccc|c} 1 & 1 & 0 & 5 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 1 \end{array} \right]$$

$$(c) \left[\begin{array}{cc|c} 1 & 0 & 15 \\ 0 & 1 & -7 \\ 0 & 0 & 0 \end{array} \right]$$

$$(g) \left[\begin{array}{cc|c} 0 & 0 & 0 \\ 1 & 0 & 3 \\ 0 & 1 & 8 \end{array} \right]$$

$$(d) \left[\begin{array}{ccc|c} 1 & 0 & 0 & 5 \\ 0 & -1 & 0 & -3 \\ 0 & 0 & 1 & 8 \end{array} \right]$$

4. For the next two word problems do the following:

I) Define the variables that are used in setting up the system of equations.

II) Set up the system of equations that represents this problem. III) Solve for the solution.

IV) If the solution is parametric, then tell what restrictions should be placed on the parameter(s). Also give three specific solutions.

(a) Fred, Bob, and George are avid collectors of baseball cards. Among the three of them, they have 924 cards. Bob has three times as many cards as Fred, and George has 100 more cards than Fred and Bob do combined. How many cards do each of the friends have?

(b) In a laboratory experiment, a researcher wants to provide a rabbit with exactly 1000 units of vitamin A, exactly 1600 units of vitamin C and exactly 2400 units of vitamin E. The rabbit is fed a mixture of three foods. Each gram of food 1 contains 2 units of vitamin A, 3 units of vitamin C, and 5 units of vitamin E. Each gram of food 2 contains 4 units of vitamin A, 7 units of vitamin C, and 9 units of vitamin E. Each gram of food 3 contains 6 units of vitamin A, 10 units of vitamin C, and 14 units of vitamin E. How many grams of each food should the rabbit be fed?

5. Solve the system $\begin{cases} 5x - 3y = 7 \\ 2x + 6y = -1 \end{cases}$ using the Gauss-Jordan elimination method.

6. Solve for the variables x , y , z , and u . If this is not possible, explain why.

$$\begin{bmatrix} -1 & 0 & 5 \\ 7 & 3 & 0 \end{bmatrix} \begin{bmatrix} 7x & -2 \\ 4 & -3z \\ x & 8 \end{bmatrix} + \begin{bmatrix} y & -40x \\ -12 & 16 \end{bmatrix}^T = \begin{bmatrix} 9 & 3u \\ -6 & 0 \end{bmatrix}$$

7. Find the matrix A that makes the following equation true:

$$\begin{bmatrix} -5 & 3 \\ 8 & 7 \end{bmatrix} + \frac{1}{3} \begin{bmatrix} 2 & -1 \\ -7 & 5 \end{bmatrix} A = I_2$$

8. Use the given matrices to compute each of the following. If an operation is not possible, explain why.

$$A = \begin{bmatrix} -5 & 3 \\ 7 & 8 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 2 \\ 5 & -8 \\ 3 & 5 \end{bmatrix}, \quad C = \begin{bmatrix} 5 & 4 & 7 \\ 6 & -3 & 1 \end{bmatrix}, \quad D = 3 \times 3 \text{ nonsingular matrix.} \\ E = 4 \times 4 \text{ singular matrix.}$$

- (a) $B + C$
 (b) BC
 (c) $D^{-1}C$
 (d) $AB^T - 5C$
 (e) DD^{-1}
 (f) $E^{-1}E$
 (g) CA
 (h) C^{-1}
9. Work #41 on page 130 of Section 2.5 in your textbook by Tan. This is a problem about understanding the meaning of the entries in the product of two matrices. There is also another example in Week in Review #2.
10. First write the following system as a matrix equation and then solve by using a matrix inverse.

$$\begin{aligned} -3x &= 4y + z \\ y - 7 &= -x + 5z \\ 2x + z &= 14 - y \end{aligned}$$

11. A simple economy depends on three commodities: oil, corn, and coffee. Production of 1 unit of oil requires 0.1 units of oil and 0.2 units of corn. Producing 1 unit of corn requires 0.2 units of oil, 0.1 units of corn, and 0.05 units of coffee. To produce 1 unit of coffee, 0.1 units of oil, 0.05 units of corn, and 0.1 units of coffee are used.
- (a) Find the production level required to meet an external demand of 1,000 units of each of these three commodities. (#18, pg. 114 of *Finite Mathematics and Calculus* by Lial, Greenwell, and Ritchey)
- (b) How much of each commodity is consumed internally to meet this demand?
12. Graph the solution set for the following system of inequalities. Label all corner points. Is the solution set bounded or unbounded?
- $$\begin{aligned} x - y &\leq 0 \\ 2x + y &\leq 6 \\ 4x - y &\geq -6 \end{aligned}$$
13. Ruff, Inc. makes dog food out of chicken and grain. Chicken has 10 grams of protein and 5 grams of fat per ounce, and grain has 2 grams of protein and 2 grams of fat per ounce. A bag of dog food must contain at least 200 grams of protein and at least 150 grams of fat. If chicken costs 10 cents per ounce and grain costs 1 cent per ounce, how many ounces of each should Ruff use in each bag of dog food in order to minimize cost? **Formulate as a linear programming problem, but do not solve.**
- (#23, pg. 217 of *Finite Mathematics* by Waner and Costenoble)
14. A company makes two calculators: a business model and a scientific model. The business model contains 10 microcircuits and requires 20 minutes to program, while the scientific model contains 20 microcircuits and requires 30 minutes to program. The company has a contract that requires it to use at least 320 microcircuits each day, and the company has 14 hours of programming time available each day. The company also wants to make at least twice as many business calculators as scientific calculators. If each business calculator requires 10 production steps and each scientific calculator requires 12 production steps, how many calculators of each type should be made each day to minimize the number of production steps? **Formulate as a linear programming problem, but do not solve.**

15. Solve using the Method of Corners.

$$\begin{array}{ll} \text{Maximize} & P = 2x + 5y \\ \text{subject to} & -3x + y \leq 6 \\ & x - y \leq 7 \\ & x \geq 0 \\ & y \geq 0 \end{array}$$

16. Solve using the Method of Corners.

$$\begin{array}{ll} \text{Maximize} & P = 2x + y \\ \text{subject to} & x + y \leq 4 \\ & 2x + y \leq 5 \\ & 5x + y \leq 10 \\ & x \geq 0 \\ & y \geq 0 \end{array}$$