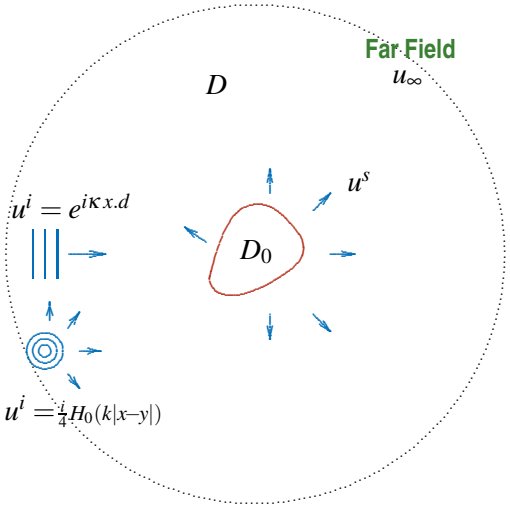
Work on Inverse Obstacle Scattering

Acoustic scattering: A plane wave $u^i = e^{i\kappa x.d}$ or a point source $\frac{i}{4}H_0(\kappa|x-y|)$ coming from infinity is fired at a cylindrical obstacle D_0 of unknown shape and location. The scattered wave u^s is measured at "infinity" – the far field pattern u_∞.



For acoustic scattering u^s satisfies the Helmholtz equation: $\triangle u + \kappa^2 u = 0$ in $D = \mathbb{R}^n \setminus D_0$, $n = 2, 3$.

D_0 can be "sound soft" (Dirichlet boundary conditions) or "sound hard" (Neumann conditions) or of impedance type $\frac{\partial u}{\partial \nu} + \gamma(x)u = 0$ where $\gamma(x)$ may also be unknown and recovered as part of the inverse problem. In the electromagnetic case we need Maxwell's equations.

Several incident waves be used, either from different directions d or of different frequencies κ. There also can be multiple simply-connected obstacles.

The literature on the subject is extensive but the question of uniquness in recovering a D_0 with a smooth (C^2 boundary) and without fixed size restrictions from a single incident wave remains a long-standing open conjecture.

Of course, there is also the situation of a penetrable obstacle D_0 with density $\rho(x)$ differing from the background medium and one seeks revovery of the internal density from multiple input waves.

The standard reference here is

> David Colton and Rainer Kress: *Inverse Acoustic and Electromagnetic Scattering Theory*, Springer Verlag, 4th edition, 2019, {doi: 10.1007/978-3-030-30351-8}.

The references below are to my publications in this area

[1] Rainer Kress and William Rundell. Inverse scattering for shape and impedance revisited. *J. Integral Equations Appl.*, 30(2):293–311, 2018.

[2] Rainer Kress and William Rundell. A nonlinear integral equation and an iterative algorithm for an inverse source problem. *J. Integral Equations Appl.*, 27(2):179–198, 2015.

[3] Rainer Kress and William Rundell. Reconstruction of extended sources for the Helmholtz equation. *Inverse Problems*, 29(3):035005, 15, 2013.

[4] Rainer Kress and William Rundell. Nonlinear integral equations and the iterative solution for an inverse boundary value problem. *Inverse Problems*, 21(4):1207–1223, 2005.

[5] Rainer Kress and William Rundell. Inverse scattering for shape and impedance. volume 17, pages 1075–1085. 2001. Special issue to celebrate Pierre Sabatier's 65th birthday (Montpellier, 2000).

[6] Rainer Kress and William Rundell. Inverse obstacle scattering using reduced data. *SIAM J. Appl. Math.*, 59(2):442–454, 1999.

[7] Rainer Kress and William Rundell. Inverse obstacle scattering with modulus of the far field pattern as data. In *Inverse problems in medical imaging and nondestructive testing (Oberwolfach, 1996)*, pages 75–92. Springer, Vienna, 1997.

[8] Rainer Kress and William Rundell. A quasi-Newton method in inverse obstacle scattering. *Inverse Problems*, 10(5):1145–1157, 1994.

[9] Gang Bao, Shuai Lu, William Rundell, and Boxi Xu. A recursive algorithm for multifrequency acoustic inverse source problems. *SIAM J. Numer. Anal.*, 53(3):1608–1628, 2015.

[10] William Rundell. Recovering an obstacle and a nonlinear conductivity from Cauchy data. *Inverse Problems*, 24(5):055015, 12, 2008.