Section 1.3: Vector functions

Parametric equations:

$$x = x(t), \quad y = y(t)$$

where the variable t is called a **parameter.** Each value of the parameter t defines a point that we can plot. As t varies over its domain we get a collection of points $(x, y) = (x(t), y(t))$ on the plane which is called the **parametric curve**.

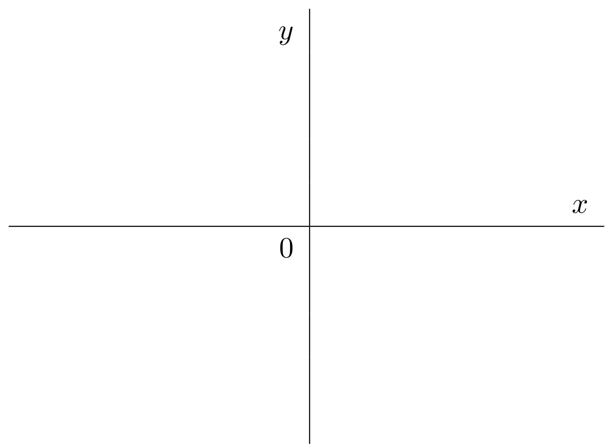
Each parametric curve can be represented as the **vector function**:

$$\overrightarrow{r(t)} = \langle x(t), y(t) \rangle .$$

Note that Parametric curves have a **direction of motion** given by increasing of parameter t. So, when sketching parametric curves we also include arrows that show the direction of motion.

EXAMPLE 1. *Examine the parametric curve* $x = \cos t, \quad y = \sin t, \quad 0 \le t \le 3\pi/2$.

t	0	$\pi/4$	$\pi/2$	$\ldots$
(x, y)				

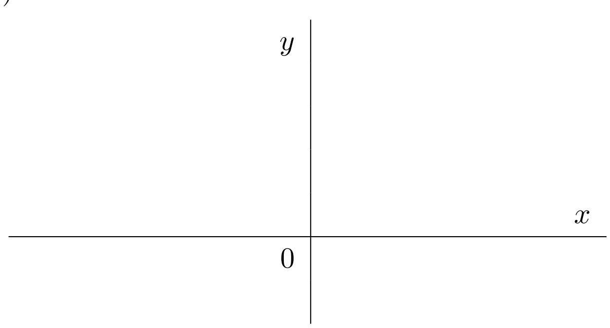


EXAMPLE 2. *Given* $\mathbf{r}(t) = \left\langle t + 1, t^2 \right\rangle$.

(a) *Does the point* $(4, 3)$ *belong to the graph of* $\mathbf{r}(t)$*?*

(b) *Sketch the graph of* $\mathbf{r}(t)$.

t	$\mathbf{r}(t)$
-2	
-1	
0	
1	
2	



(c) *Find the Cartesian equation of* $\mathbf{r}(t)$ *eliminating the parameter.*

EXAMPLE 3. *Find the Cartesian equation for* $\mathbf{r}(t) = \cos t\mathbf{i} + \cos(2t)\mathbf{j}$

EXAMPLE 4. *An object is moving in the xy-plane and its position after t seconds is given by* $\mathbf{r}(t) = \langle 1 + t^2, 1 + 3t\rangle$.

(a) *Find the position of the object at time* $t = 0$.

(b) *At what time does the object reach the point* $(10, 10)$.

(c) *Does the object pass through the point* $(20, 20)$*?*

(d) *Find an equation in x and y whose graph is the path of the object.*

A **Vector equation of the line** passing through the point (x_0, y_0) and parallel to the vector $\mathbf{v} = \langle a, b\rangle$ is given by

$$\mathbf{r} = \mathbf{r_0} + t\mathbf{v},$$

where $\mathbf{r_0} = \langle x_0, y_0\rangle$.

The vector equation of the line can be rewritten in parametric form. Namely, we have

$$\begin{aligned}\langle x(t), y(t)\rangle &= \mathbf{r} = \mathbf{r_0} + t\mathbf{v} = \\ &= \langle x_0, y_0\rangle + t\langle a, b\rangle = \langle x_0, y_0\rangle + \langle ta, tb\rangle = \\ &= \langle x_0 + ta, y_0 + tb\rangle.\end{aligned}$$

This immediately yields that the **parametric equations of the line** passing through the point (x_0, y_0) and parallel to the vector $\mathbf{v} = \langle a, b\rangle$ are

$$x(t) = x_0 + at, \quad y(t) = y_0 + bt.$$

EXAMPLE 5. *Find parametric equations of the line*

(a) *passing through the point* $(1, 0)$ *and parallel to the vector* $\mathbf{i} - 4\mathbf{j}$*;*

(b) *passing through the point* $(-4, 5)$ *with slope* $\sqrt{3}$*;*

(c) *passing through the points* $(7, 2)$ *and* $(3, 2)$*.*

EXAMPLE 6. *Determine whether the lines* $\mathbf{r}(t) = \langle 1 + t, 1 - 3t\rangle$, $\mathbf{R}(s) = \langle 1 + 3s, 12 + s\rangle$ *are parallel, orthogonal or neither. If they are not parallel, find the intersection point.*