3.6: Implicit differentiation

EXAMPLE 1. *Find y' if the $y = y(x)$ satisfies the equation $xy = 5$ for all values of x in its domain and evaluate $y'(5)$.*

`Solution 1` (by explicit differentiation):

`Solution 2` (by implicit differentiation):

EXAMPLE 2. **(a)** *If $x^2 + y^2 = 16$ find* $\dfrac{\mathrm{d}y}{\mathrm{d}x}$.

(b) *Find the equation of the tangent line to $x^2 + y^2 = 16$ at the point $(2, 2\sqrt{3})$.*

EXAMPLE 3. *Find* $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ *for the following:*

(a) $4x^3 + 2y^2 = 4xy^5 + y$

(b) $x^3 - \cot(xy^2) = x\cos y$

(c) $(x^2 + y^2)^5 = x^2y^3$

DEFINITION 4. *Two curves are said to be* **orthogonal** *if at the point(s) of their intersection, their tangent lines are orthogonal(perpendicular). In this case we also say that the angle between these curves is* $\frac{\pi}{2}$.

`Illustration:` Consider two families of curves:

$$x^2 + y^2 = r^2, \quad y = kx,$$

where r and k are real parameters.

EXAMPLE 5. *Are these curves orthogonal?*

$$x^2 - y^2 = 5, \quad 4x^2 + 9y^2 = 72$$

EXAMPLE 6. *Find the equations of both the tangent lines to the ellipse* $x^2 + 4y^2 = 36$ *that pass through the point* $(12, 3)$.