4.8:Indeterminate forms and L'Hospital's Rule

Indeterminate forms: Consider

$$\lim_{x\to a}\frac{f(x)}{g(x)}. \tag{1}$$

- If both $f(x)\to 0$ and $g(x)\to 0$ as $x\to a$, then (1) is called an **indeterminate form of type $\frac{0}{0}$.**
- If both $f(x)\to \pm\infty$ and $g(x)\to \pm\infty$ as $x\to a$, then (1) is called an **indeterminate form of type $\frac{\infty}{\infty}$.**

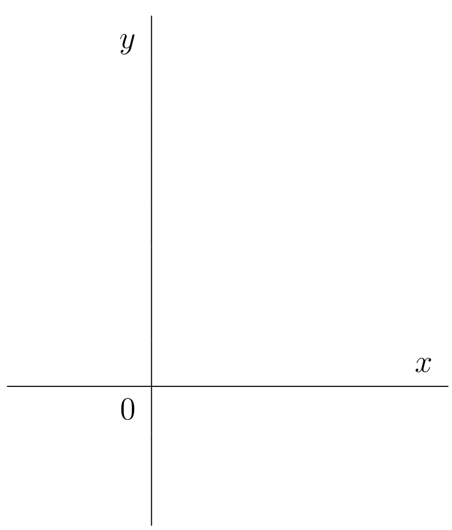
EXAMPLES:

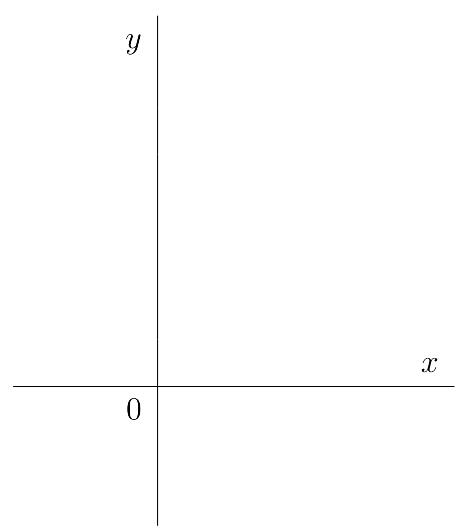
$\lim_{x\to 0}\frac{\sin x}{x} = \underline{\qquad}$, $\lim_{x\to 1}\frac{x-x^2}{x^2-1} = \underline{\qquad}$, $\lim_{x\to \infty}\frac{\ln x}{x^3} = \underline{\qquad}$,

L'HOSPITAL'S RULE: Suppose f and g are differentiable and $\lim_{x\to a}\frac{f(x)}{g(x)} = \frac{0}{0}$ or $\frac{\infty}{\infty}$ then

$$\lim_{x\to a}\frac{f(x)}{g(x)} = \lim_{x\to a}\frac{f'(x)}{g'(x)}$$

if the limit on the right side exists (or is ∞ or $-\infty$).





EXAMPLE 1. *Evaluate each of the following limits:*

(a) $\lim_{x\to 0} \frac{\sin x}{x}$

(b) $\lim_{x\to\infty} \frac{x^2}{e^x}$

(c) $\lim_{x\to 0} \frac{\sin x - x}{x^3}$

Indeterminate form of type $0 \cdot \infty$: $\lim_{x\to a} f(x)g(x)$

Write the product fg as a quotient to get an indeterminate form of type $\frac{0}{0}$ or $\frac{\infty}{\infty}$:

EXAMPLE 2. *Evaluate each of the following limits:*

(a) $\lim_{x\to\infty} x\ln(1+\frac{1}{x})$

(b) $\lim_{x\to-\infty} xe^x$

(c) $\lim_{x\to\pi/4}(1-\tan x)\sec(2x)$

Indeterminate form of type $\infty-\infty$: $\lim_{x\to a}(f(x)-g(x))$

Try to convert the difference into a quotient to get an indeterminate form of type $\frac{0}{0}$ or $\frac{\infty}{\infty}$.

EXAMPLE 3. *Find:* $\lim_{x\to 1}\left(\frac{1}{\ln x}-\frac{1}{x-1}\right)$

Indeterminate form of type 0^0, ∞^0, 1^∞: $\lim_{x\to a} f(x)^{g(x)}$

Write the function as an exponential $0\cdot\infty$. It leads to an indeterminate form of type $0\cdot\infty$.

EXAMPLE 4. *Find the following limits:*

(a) $\lim_{x\to\infty} x^{\frac{1}{x}} =$

(b) $\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x =$