# 5.5: Applied Maximum and Minimum Problems

OPTIMIZATION PROBLEMS

**First derivative test for absolute extrema**: Suppose that $c$ is a critical number of a continuous function $f$ defined on an interval.

- If $f'(x) > 0$ for all $x < c$ and $f'(x) < 0$ for all $x > c$, then $f(c)$ is the absolute maximum value of $f$.
- If $f'(x) < 0$ for all $x < c$ and $f'(x) > 0$ for all $x > c$, then $f(c)$ is the absolute minimum value of $f$.

Alternatively,

- *If $f''(x) < 0$ for all $x$ (so $f$ is always concave downward) then the local maximum at c must be an absolute maximum.*
- *If $f''(x) > 0$ for all $x$ (so $f$ is always concave upward) then the local minimum at c must be an absolute minimum.*

EXAMPLE 1. *A rectangular storage container with an open top is to have a volume of* $10\text{m}^3$. *The length of its base is twice the width. Material for the base costs* \$10 *per squire meter. Material for the sides costs* \$6 *per square meter. Find the cost of materials for the cheapest such container.*

EXAMPLE 2. *A rectangle is bounded by the $x$-axis and the semicircle $y = \sqrt{1 - x^2}$. What length and width should the rectangle have so that its area is a maximum? (*Equivalently,find the dimensions of the largest rectangle that can be inscribed in the semi-disk with radius 1. *)*