Math 220 – Homework 6

Due Thursday 10/20 at the beginning of class

PART A

Problems from the textbook:

Section 2.1 # 1(b, c, e, i); 2(b, f,h); 4(b,c,f,i); 5; 14; 15;

Section 2.2 # 4(b, e), 5(b, e), 6

Section 2.3 # 2, 4(a,b,c).

PART B

1. Determine the truth or falsehood of the following statements. (Write TRUE or FALSE for each statement.)
   (a) The contrapositive of the statement
   "*If all elements of A are elements of B, then A is a subset of B*"
   is the statement
   "*If A is a subset of B, then all elements of A are elements of B*".
   (b) $\{a, b\} = \{b, a, b\}$
   (c) If $A = \{m \in \mathbb{Z} |\ 2 < m \leq 5\}$ then $|A| = 4$.
   (d) The empty set is a subset of every set except itself.
   (e) $5 \in \{\{-1, 5\}, \{-5, 2017, 0\}, \{1, 2\}\}$.
   (f) If $A = \{a, \{a, b, c\}\}$ and $B = \{\{c, d\}, \{a, b, c, d\}\}$ then $|A| = |B|$. .
2. For the sets $A = \{x \in \mathbb{Z} |\ 2 \leq x < 4\}$ and $B = \left\{x \in \mathbb{R} | x^4 = 1\right\}$ form the following Cartesian products:
   (a) $B \times A$
   (b) $B \times A \times B$.
3. Let A, B, and C be nonempty subsets of a universal set U. Disprove the following statements:
   (a) If $A \cap B = A \cap C$, then $B = C$.
   (b) If $A - B = C - B$, then implies $A = C$.
   (c) If A is not a subset of B and B is not a subset of A, then $A \cap B = \emptyset$.
4. Let $U = \mathbb{R}$ be a universal set. Consider $A = \{x \in \mathbb{R} |\ |2x + 3| \geq 29\}$ and $B = \{x \in \mathbb{R} |\ |x| \leq 1\}$.
   (a) Express the sets A and B using interval notation (as an interval or a union of intervals).
   (b) Determine $\overline{A} \cap \overline{B}$ as an interval or a union of intervals.
5. Let $U = \{x, y, a, b, c\}$ be the universal set and let $M = \{x, y, a, b\}$, $N = \{a, c, x, y\}$, $P = \{b, c\}$. Determine the following (show all intermediate steps):
   (a) $\overline{M} \cup (N \cap P)$
   (b) $\overline{\overline{P \cup N} \cap U}$
   (c) $\overline{(M \cup P) - (N \cap P)}$
6. Let $A = \{(x, y) \in \mathbf{Z}^+ \times \mathbf{Z} |\ |x| + |y| = 2\}$. List all elements of A and find $|A|$.