Appendix D: Trigonometry

Angles. Angles can be measured in degrees or radians.

Everything in most calculus classes will be done in radians.

The angle given by a complete revolution contains 360^0, or 2π radians, i.e. $360^0 = 2\pi$ rad. Hence,

$$\pi \text{ rad} = 180^0$$

and

$$1\text{rad} = \left(\frac{180}{\pi}\right)^0, \quad 1^0 = \frac{\pi}{180}\text{rad}.$$

EXAMPLE 1. *Convert* -315^0 *to radians.*

EXAMPLE 2. *Convert* $\dfrac{5\pi}{6}$ *to degrees.*

For basic angles we have the following table (Know it!)

Degree	0	30	45	60	90	180	270	360
Radians	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π

Six trigonometric functions and how they relate to each other.

$$\cos x \qquad \sin x$$

$$\tan x = \frac{\sin x}{\cos x} \qquad \cot x = \frac{\cos x}{\sin x} = \frac{1}{\tan x}$$

$$\sec x = \frac{1}{\cos x} \qquad \csc x = \frac{1}{\sin x}$$

For an acute angle θ all the trig functions can be defined as ratios of sides of a right triangle:

$$\cos\theta = \frac{\text{adj}}{\text{hyp}} \qquad \sin\theta = \frac{\text{opp}}{\text{hyp}}$$
$$\tan\theta = \frac{\text{opp}}{\text{adj}} \qquad \cot\theta = \frac{\text{adj}}{\text{opp}}$$
$$\sec\theta = \frac{\text{hyp}}{\text{adj}} \qquad \csc\theta = \frac{\text{hyp}}{\text{opp}}$$

Pythagorean Theorem

The exact trigonometric ratios for certain angles can be read from the following two special triangles.

$$\sin\frac{\pi}{4} = \frac{1}{\sqrt{2}} \qquad \sin\frac{\pi}{6} = \frac{1}{2} \qquad \sin\frac{\pi}{3} =$$

$$\cos\frac{\pi}{4} = \frac{1}{\sqrt{2}} \qquad \cos\frac{\pi}{6} = \qquad \cos\frac{\pi}{3} =$$

$$\tan\frac{\pi}{4} = \qquad \tan\frac{\pi}{6} = \frac{1}{\sqrt{3}} \qquad \tan\frac{\pi}{3} =$$

Using of unit circle

EXAMPLE 3. *Evaluate each of the following:*

(a) $\sin \dfrac{5\pi}{6}$

(b) $\sin\left(-\dfrac{5\pi}{6}\right)$

(c) $\cos\left(\dfrac{4\pi}{3}\right)$

(d) $\cos\left(-\dfrac{4\pi}{3}\right)$

(e) $\tan\left(-\dfrac{\pi}{4}\right)$

EXAMPLE 4. *If* $\sin\theta = \dfrac{2}{3}$ *and* $0 < \theta < \pi/2$, *find the other five trigonometric functions of* θ.

Know these identities

$$\sin^2\theta + \cos^2\theta = 1 \qquad \tan^2 + 1 = \sec^2\theta$$

$$\sin 2\theta = 2\sin\theta\cos\theta \qquad \cos 2\theta = 2\cos^2\theta - 1$$

$$\cos^2\theta = \frac{1}{2}(1 + \cos 2\theta) \qquad \sin^2\theta = \frac{1}{2}(1 - \cos 2\theta)$$

EXAMPLE 5. *Find all values of* x *in the interval* $[0, 2\pi]$ *such that*

(a) $\sin 2x = \cos x$

(b) $\sin x = \dfrac{3}{2}$

EXAMPLE 6. *If* $\sec y = \dfrac{5}{4}$, *where* $0 < y < \pi/2$, *evaluate* $\sin 2y$.

Graphs of the trig functions

- $y = \sin x$

- $y = \cos x$

- $y = \tan x$

EXAMPLE 7. *Graph the following functions:*

(a) $f(x) = \sin(x + \frac{\pi}{4})$

(b) $f(x) = 2 - \cos x$