Name_____

There are 5 questions, for a total of 100 points. Point values are written beside each question. Calculators may be used only for basic arithmetic operations. *Show your work for full credit.*

1. (a) [5] State the definition of *derivative*, that is $f'(x) =$

    (b) [15] If $f(x) = x^2 + 2x$, find $f'(x)$ using the definition of derivative.

2. Differentiate the following functions.

(a) [10] $f(x) = \dfrac{\sqrt[3]{x}}{1 - x^2}$

(b) [10] $f(x) = \sec^3(2x + 1)$

3. [20] Find an equation of the line tangent to the curve $y^2 - x^2y = \cos(x)$ at the point $\left(\dfrac{\pi}{2}, 0\right)$.

4. The position of a particle in the plane is $\mathbf{r}(t) = \langle \cos(2t), \ t^2 - 1 \rangle$ at time t.

(a) [12] Find the velocity of the particle at time $t = \dfrac{\pi}{4}$.

(b) [8] Find the speed of the particle at time $t = \dfrac{\pi}{4}$.

5. [20] A video camera is placed 1500 m from the base of a rocket launching pad. A rocket rises vertically and its speed is 300 m/s when it has risen 2000 m. If the video camera is always kept focused on the rocket, how fast is the camera's angle of elevation changing at that moment?