MATH 171 EXAM 3 2014 PARTIAL SOLUTIONS

1. (a) $f'(x) = x \cdot \frac{1}{x^2+1} \cdot 2x + \ln(x^2+1) = \frac{2x^2}{x^2+1} + \ln(x^2+1)$

(b) $f'(x) = \frac{1}{\sqrt{1-(2x-1)^2}} \cdot 2 = \frac{2}{\sqrt{1-(4x^2-4x+1)}} = \frac{2}{\sqrt{4x-4x^2}} = \frac{1}{\sqrt{x-x^2}}$

2. (a) By L'Hospital's Rule, $\lim_{x\to 0} \frac{\tan^{-1}x}{x} = \lim_{x\to 0} \frac{\frac{1}{1+x^2}}{1} = 1$

(b) Apply L'Hospital's Rule twice, after rewriting:
$\lim_{x\to -\infty} x^2 e^x = \lim_{x\to -\infty} \frac{x^2}{e^{-x}} = \lim_{x\to -\infty} \frac{2x}{-e^{-x}} = \lim_{x\to -\infty} \frac{2}{e^{-x}} = 0$

3. $f'(x) = 1 - \sec^2 x$, set $1-\sec^2 x = 0$ to obtain $\sec^2 x = 1$ or $\sec x = \pm 1$, so
$x = 0$ (other solutions are not in the interval $[-\frac{\pi}{4}, \frac{\pi}{4}]$). Evaluate:
$f(0) = 0$ $\quad f(\frac{\pi}{4}) = \frac{\pi}{4} - 1 \approx -.215 \quad f(-\frac{\pi}{4}) = -\frac{\pi}{4}+1 \approx .215$
Absolute maximum function value: $.215$
Absolute minimum function value: $-.215$

4. (a) $f'(x) = 3x^2 - 9$, set $3x^2-9=0$ to obtain $x = \pm\sqrt{3}$

$\underbrace{\quad}_{f'>0} -\sqrt{3} \underbrace{\quad}_{f'<0} \sqrt{3} \underbrace{\quad}_{f'>0}$

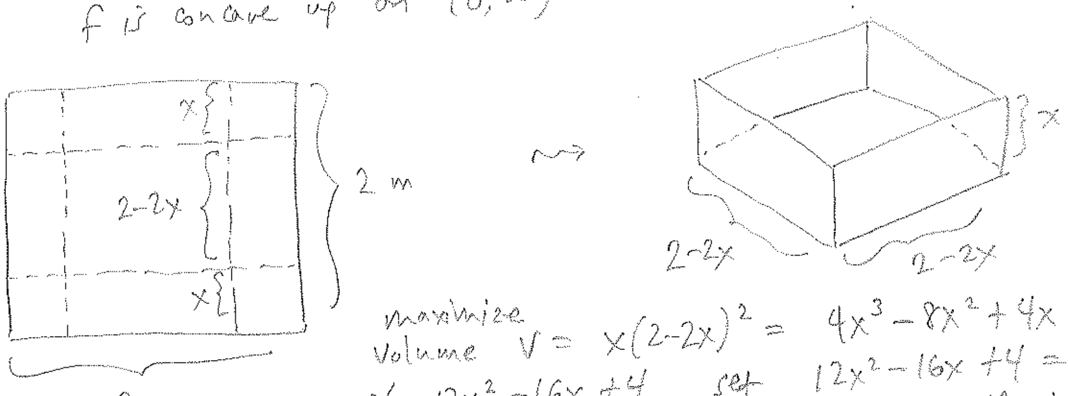
f is increasing on $(-\infty, -\sqrt{3})$, $(\sqrt{3}, \infty)$
f is decreasing on $(-\sqrt{3}, \sqrt{3})$

(b) $f''(x) = 6x$, set $6x = 0$ to obtain $x=0$

$\underbrace{\quad}_{f''<0}\ 0\ \underbrace{\quad}_{f''>0}$

f is concave down on $(-\infty, 0)$
f is concave up on $(0, \infty)$

5.



maximize
Volume $V = x(2-2x)^2 = 4x^3 - 8x^2 + 4x$
$V' = 12x^2 - 16x + 4$, set $12x^2-16x+4 = 0$ to obtain $x = \frac{1}{3}, 1$
(Note $x=1$ yields a box with 0 volume, so this does not yield a maximum.)
Considering $V' = 4(3x-1)(x-1)$, the first derivative test shows that
$x = \frac{1}{3}$ yields the maximum volume: $V = \frac{1}{3}(2-2(\frac{1}{3}))^2 = \frac{16}{27}\ m^3 \approx .593\ m^3$

6. $f(x) = 3\ln|x| + \frac{2}{3}x^{3/2} + C$
$3 = f(e) = 3\ln e + \frac{2}{3}e^{3/2} + C = 3 + \frac{2}{3}e^{3/2} + C$, so $C = -\frac{2}{3}e^{3/2}$

$f(x) = 3\ln|x| + \frac{2}{3}x^{3/2} - \frac{2}{3}e^{3/2}$