MATH 171H EXAM 3 2014 PARTIAL SOLUTIONS

1. (a) $\frac{dy}{dx} = \dfrac{x \cdot \frac{1}{1+x^2} - \tan^{-1}x}{x^2} = \dfrac{x - (1+x^2)\tan^{-1}x}{x^2(1+x^2)}$ or $\dfrac{1}{x(1+x^2)} - \dfrac{\tan^{-1}x}{x^2}$

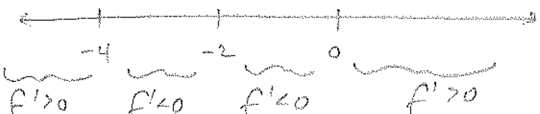
(b) $\ln y = \ln(x^{\ln x}) = (\ln x)(\ln x) = (\ln x)^2$

$\frac{1}{y}y' = (2\ln x)\left(\frac{1}{x}\right)$ so $y' = (2\ln x)\left(\frac{1}{x}\right)y = \dfrac{2\ln x}{x}x^{\ln x}$ or $2x^{\ln x - 1}\ln x$

2. (a) By L'Hospital's Rule, $\lim\limits_{x\to 0}\dfrac{1-\cos x}{\tan x} = \lim\limits_{x\to 0}\dfrac{\sin x}{\sec^2 x} = \dfrac{0}{1} = 0$

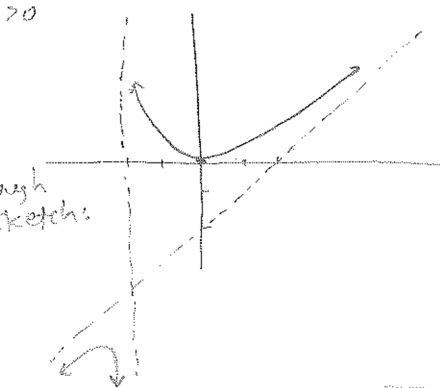
(b) By L'Hospital's Rule, $\lim\limits_{x\to 0}\dfrac{2x - \sin^{-1}x}{x+\sin^{-1}x} = \lim\limits_{x\to 0}\dfrac{2 - \frac{1}{\sqrt{1-x^2}}}{1+\frac{1}{\sqrt{1-x^2}}} = \dfrac{2-1}{1+1} = \dfrac{1}{2}$

3. $f'(x) = \dfrac{(x+2)(2x) - x^2}{(x+2)^2} = \dfrac{x(x+4)}{(x+2)^2}$

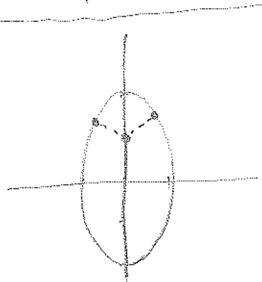


f is increasing on $(-\infty,-4)$, $(0,\infty)$
f is decreasing on $(-4,-2)$, $(-2,0)$

rough sketch:



4.



Square of distance:

$S = (x-0)^2 + (y-1)^2$

$S = x^2 + (y-1)^2$, substitute $x^2 = \dfrac{4-y^2}{4}$

$S = \dfrac{4-y^2}{4} + y^2 - 2y + 1$

$S = \frac{3}{4}y^2 - 2y + 2$

$S' = \frac{3}{2}y - 2$, set $\frac{3}{2}y - 2 = 0$ to obtain $y = \frac{4}{3}$, $x^2 = \dfrac{4-\left(\frac{4}{3}\right)^2}{4} = \dfrac{5}{9}$

Answer: $\left(\frac{\sqrt5}{3}, \frac{4}{3}\right)$, $\left(\frac{-\sqrt5}{3}, \frac{4}{3}\right)$

5. (a) F. Counterexample: $f(x) = |x|$ has a local minimum at $x=0$, but $f'(0)$ does not exist.

(b) T. By hypothesis, f is differentiable on $[0,1]$, so we may apply the Mean Value Theorem: There is a c in $(0,1)$ such that $f'(c) = \dfrac{f(1) - \overset{0}{f(0)}}{1} = f(1)$ (since $f(0)=0$). Since $0 \le f'(x) \le 1$ for all x in $[0,1]$, we have $0 \le f'(c) \le 1$, i.e. $0 \le f(1) \le 1$.

(c) T. Since f and g are concave upward, $f''(x) > 0$ and $g''(x) > 0$ for all x in I. Then $(f+g)''(x) = f''(x) + g''(x) > 0$ for all x in I, and it follows that $f+g$ is concave upward on I.

(d) F. Counterexample: $f(x) = x^2$, $g(x) = x^2 + 1$