Name_____

There are 5 questions, and a total of 100 points. Point values are written beside each question. Calculators may be used only for basic arithmetic operations. *Show your work for full credit.*

1. Find $\dfrac{dy}{dx}$:

(a) [10 points] $y = \dfrac{\tan^{-1} x}{x}$

(b) [10] $y = x^{\ln x}$

2. Find the limits:

(a) [10] $\displaystyle\lim_{x \to 0} \frac{1 - \cos x}{\tan x}$

(b) [10] $\displaystyle\lim_{x \to 0} \frac{2x - \sin^{-1} x}{x + \sin^{-1} x}$

3. [20] Find the the intervals on which $f(x) = \dfrac{x^2}{x+2}$ is increasing or decreasing.

$\dfrac{x^2}{x+2}$

4. [20] Find the points on the ellipse $4x^2 + y^2 = 4$ that are closest to the point $(0, 1)$.

5. **True/False/Justify**. Determine whether each statement is true or false, and write "T" if true, and "F" if false, in the blank. Briefly **justify** each true statement with one or two sentences explaining why it is true. (For example, if it follows from a theorem we covered, give either a rough statement of the theorem or its name, if it has a name, and explain how it applies. If it follows from a definition or formula, give the definition or formula and show how it applies.) For each false statement, give a **counterexample** to show that it is false.

_____ (a) [5] If f has a local maximum or minimum value at c, then $f'(c) = 0$.

_____ (b) [5] If $f(0) = 0$ and $0 \leq f'(x) \leq 1$ for all x in the interval $[0, 1]$, then $0 \leq f(1) \leq 1$.

5. (continued)

_____ (c) [5] If f and g are twice differentiable functions that are concave upward on an interval I, then $f + g$ is concave upward on I.

_____ (d) [5] If $f'(x) = g'(x)$ for all x in an interval (a, b), then $f(x) = g(x)$ for all x in (a, b).