Math 171H Final Exam
December 16, 2014
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Name_____

There are 10 questions, totaling 200 points. Point values are written beside each question. Calculators may be used only for basic arithmetic operations. *Show your work for full credit.*

1. (a) [5 points] State the definition of *limit*, that is $\lim_{x \to a} f(x) = L$ means

(b) [20] Prove $\lim_{x \to 2}(x^2 + 1) = 5$ using the definition of limit.

2. Evaluate the limits.

(a) [15] $\lim_{x \to 3} \dfrac{x-3}{\sqrt{x^2-9}}$

(b) [15] $\lim_{x \to 0^+} x \ln(x^2)$

3. (a) [5] State the definition of *derivative*, that is $f'(x) =$

(b) [20] If $f(x) = \dfrac{1}{\sqrt{x}}$, find $f'(x)$ using the definition of derivative.

4. [20] Let $f(x) = \begin{cases} x^2 \cos(\frac{1}{x}), & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$.

Find $f'(x)$ for all real numbers x. (Note that $f'(0)$ must be computed separately.)

5. [15] Find the x-coordinates of all points on the graph of $y = \cos(2x) - 2\cos(x)$ at which the tangent line is horizontal.

6. [15] Let f be a function for which $f(0) = 0$ and $f'(x) \geq 2$ for all x in the interval $[0, 1]$. Use the Mean Value Theorem to show that $f(1) \geq 2$.

7. [20] A ladder 3 m long rests against a vertical wall. If the bottom of the ladder slides away from the wall at a speed of 1 m/s, how fast is the angle between the top of the ladder and the wall changing when the angle is $\frac{\pi}{6}$ radians?

8. Evaluate the integrals.

(a) [15] $\int_{-3}^{3} \sqrt{9-x^2}\, dx$

(b) [15] $\int_{0}^{\frac{3}{2}} \frac{\sin^{-1}\left(\frac{x}{3}\right)}{\sqrt{9-x^2}}\, dx$

9. [10] Evaluate the following limit by recognizing it as an integral:

$$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{i}{n^2}$$

10. [10] If f is a continuous function and $\int_0^9 f(x)dx = 10$, find $\int_0^3 xf(x^2)dx$.