MATH 171H FALL 2015 EXAM 2 PARTIAL SOLUTIONS

1. (a) $\lim_{x\to 0} \frac{\sin(2x)}{\sin(x)} = \lim_{x\to 0} \frac{2\sin x\cos x}{\sin x} = \lim_{x\to 0}(2\cos x) = 2$

(b) $\lim_{x\to\infty} \frac{2e^{2x}}{2e^x + e^{2x}} \cdot \frac{\frac{1}{e^{2x}}}{\frac{1}{e^{2x}}} = \lim_{x\to\infty} \frac{2}{\frac{2}{e^x}+1} = \frac{2}{0+1} = 2$

(c) $\lim_{x\to 1}\left(\log_{\sqrt{2}}(x^2-1) - \log_{\sqrt{2}}(x-1)\right) = \lim_{x\to 1} \log_{\sqrt{2}}\left(\frac{x^2-1}{x-1}\right) = \lim_{x\to 1}\log_{\sqrt{2}}(x+1) = \log_{\sqrt{2}}(2) = 2$

2. (a) $f'(x) = \frac{(x^2+1)(2x) - x^2(2x)}{(x^2+1)^2} = \frac{2x}{(x^2+1)^2}$

(b) $f'(x) = 2\sec(xe^x)\cdot(\sec(xe^x))'$

$= 2\sec(xe^x)\cdot\left(\sec(xe^x)\tan(xe^x)\right)\cdot(xe^x)'$

$= 2\sec(xe^x)\sec(xe^x)\tan(xe^x)(xe^x+e^x)$

$= 2e^x(x+1)\sec^2(xe^x)\tan(xe^x)$

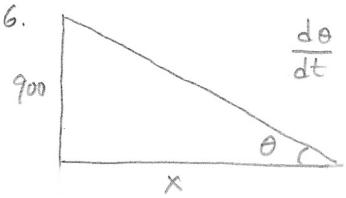
3. First consider $g(x) = \tan x$, whose derivative is $g'(x) = \sec^2 x$. When $g(x) = 0$, i.e. when $x = k\pi$ $(k\in\mathbb{Z})$, $g'(x) = \sec^2(k\pi) = (\sec(k\pi))^2 = (\pm 1)^2 = 1$. This fact and examination of the graph show that $f(x) = |\tan x|$ is not differentiable at those values. It is also not differentiable where it is not defined, e.g. when $x = \pm\frac{\pi}{2} + 2k\pi$ $(k\in\mathbb{Z})$. Combining these two expressions, we see that f is not differentiable at $x = k\frac{\pi}{2}$ $(k\in\mathbb{Z})$.

4. Note that the given point $(1,0)$ results from taking $t=0$.
$r'(t) = \langle e^t, 2\cos t\rangle$ so $r'(0) = \langle 1, 2\rangle$. The slope is $\frac{2}{1} = 2$.
The tangent line is $y - 0 = 2(x-1)$, or $y = 2x-2$.

5. Note that $g(3) = 0$ since $f(0) = 3$, and $f'(x) = 1 + \frac{\pi}{2}\sec\left(\frac{\pi}{2}x\right)$.

$g'(3) = \frac{1}{f'(g(3))} = \frac{1}{f'(0)} = \frac{1}{1+\frac{\pi}{2}\sec(0)} = \frac{1}{1+\frac{\pi}{2}} = \frac{2}{2+\pi}$

6.



$\frac{d\theta}{dt} = -\frac{1}{4}$ rad/h

$\frac{dx}{dt} = ?$ when $\theta = 30° = \frac{\pi}{6}$

When $\theta = \frac{\pi}{6}$:

$\tan\frac{\pi}{6} = \frac{900}{x}$

$\frac{1}{\sqrt{3}} = \frac{900}{x}$

$x = 900\sqrt{3}$

$\sec^2\frac{\pi}{6} = \frac{1}{\cos^2\frac{\pi}{6}} = \frac{1}{\left(\frac{\sqrt{3}}{2}\right)^2} = \frac{4}{3}$

$\tan\theta = \frac{900}{x}$

$\sec^2\theta\,\frac{d\theta}{dt} = -\frac{900}{x^2}\,\frac{dx}{dt}$

$\frac{4}{3}\cdot\left(-\frac{1}{4}\right) = -\frac{900}{900^2\cdot 3}\cdot\frac{dx}{dt}$

$\frac{dx}{dt} = 900$ ft/h

7. $y = \frac{c}{x}$

$y' = -\frac{c}{x^2}$, at a point (x_0, y_0) for which $x_0 y_0 = c$,

the tangent line therefore has slope $m = -\frac{c}{x_0^2}$

and equation $y - y_0 = -\frac{c}{x_0^2}(x - x_0)$

$y = -\frac{c}{x_0^2}x + \frac{c}{x_0} + y_0 = -\frac{c}{x_0^2}x + 2y_0$ (since $y_0 = \frac{c}{x_0}$)

The y-intercept is $2y_0$.

By symmetry, the x-intercept is $2x_0$.

Their product is $(2y_0)(2x_0) = 4x_0y_0 = 4c$

(since $x_0y_0 = c$)