MATH 171H FALL 2015 EXAM 3 SOLUTIONS

1. (a) $\frac{dy}{dx} = (1+x^2)\cdot\frac{1}{1+x^2} + 2x\tan^{-1}x = 1 + 2x\tan^{-1}x$

(b) $\frac{dy}{dx} = \frac{\sqrt{x}\cdot\frac{1}{x} - \ln x\cdot\frac{1}{2\sqrt{x}}}{x}\cdot\frac{2\sqrt{x}}{2\sqrt{x}} = \frac{2-\ln x}{2x\sqrt{x}}$

2. (a) $\lim_{x\to\infty}\frac{\ln x}{e^x} = \lim_{x\to\infty}\frac{\frac{1}{x}}{e^x} = \lim_{x\to\infty}\frac{1}{xe^x} = 0$ (by applying L'Hospital's Rule)

(b) $\lim_{x\to\frac{\pi}{2}}\left(\sec(3x)\sin(4x)\right) = \lim_{x\to\frac{\pi}{2}}\frac{\sin(4x)}{\cos(3x)} = \lim_{x\to\frac{\pi}{2}}\frac{4\cos(4x)}{-3\sin(3x)} = \frac{4}{3}$ (L'Hospital's Rule)

3. By the Mean Value Theorem, there is a c in $(0,1)$ for which

$$f'(c) = \frac{f(1)-f(0)}{1-0} = f(1) \quad (\text{since } f(0)=0).$$

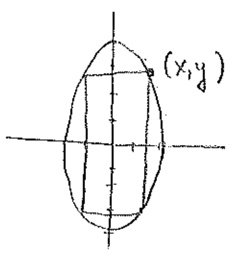
Since $f'(x)\geq 2$ for all x in the interval, we have $f'(c)\geq 2$. Since $f(1) = f'(c)$, it follows that $f(1)\geq 2$.

4. $f(x) = \ln|x| + \tan x + C$

$0 = f(\pi) = \ln|\pi| + \tan\pi + C = \ln\pi + C$, so $C = -\ln\pi$

$f(x) = \ln|x| + \tan x - \ln\pi$

5.



Let (x,y) be a point in the first quadrant on the ellipse, so $4x^2+y^2=16$.

area $A = (2x)(2y) = 4xy = 4x\sqrt{16-4x^2}$

$$\frac{dA}{dx} = 4x\cdot\frac{1}{2\sqrt{16-4x^2}}(-8x) + 4\sqrt{16-4x^2}$$

$$= \frac{-16x^2 + 4(16-4x^2)}{\sqrt{16-4x^2}} = \frac{64-32x^2}{\sqrt{16-4x^2}}$$

Setting $\frac{dA}{dx}=0$, we find $64-32x^2=0$, so $x=\sqrt{2}$. The values of x we consider lie in the interval $[0,2]$, and this value of x will yield an absolute maximum value of A. (The endpoints give a value of 0 for A.)

maximum area: $A = 4\sqrt{2}\sqrt{16-4(\sqrt{2})^2} = 4\sqrt{2}\sqrt{8} = 4\cdot 4 = 16$

6. (a) F. Counterexample: $f(x) = x^3$

$f'(x) = 3x^2$ so $f'(0) = 0$ but f does not have a local maximum, nor a local minimum, at $x = 0$.

(b) F. Counterexample: $f(x) = x$, $g(x) = x$, $a = -1$, $b = 0$

Both f and g are increasing on $[-1, 0]$, but $(fg)(x) = x^2$ is decreasing on $[-1, 0]$.

(Note $(fg)'(x) = f'(x)g(x) + g'(x)f(x)$, and since f, g take negative values on $[-1, 0]$, we also see that the derivative of fg takes negative values on $[-1, 0]$.)

(c) F. Counterexample: $f(x) = x^4$

$f'(x) = 4x^3$

$f''(x) = 12x^2$

$f''(0) = 0$ but f does not have an inflection point at $x = 0$

(d) T. Since $f'(x) = g'(x)$, we have $f(x) = g(x) + C$.

Now $f(a) = g(a)$ and $f(a) = g(a) + C$, which implies $C = 0$.

So $f(x) = g(x)$.