Math 171 Practice Final Exam

1. Find the cosine of the angle between the lines given by $\mathbf{r}_1(t) = \langle 2t - 2, \ -t + 1\rangle$ and $\mathbf{r}_2(t) = \langle 3t + 1, \ 2t + 5\rangle$.

2. Find the following limits.

   (a) $\lim\limits_{x \to 4} \dfrac{4 - x}{2 - \sqrt{x}}$

   (b) $\lim\limits_{x \to 0} x^4 \sin\left(\dfrac{2}{x}\right)$

3. (a) State the definition of *derivative*, that is $f'(x) =$

   (b) If $f(x) = \sqrt{x}$, find $f'(x)$ using the definition of derivative.

4. Find the equation of the line tangent to the graph of $\mathbf{r}(t) = \langle \sin^{-1} t, \ e^{\pi t}\rangle$ at the point $(0, 1)$.

5. For each function f, find its derivative f'.

   (a) $f(x) = x\cos(x^2 - 1)$

   (b) $f(x) = \dfrac{\sqrt{x} + 1}{\ln x}$

   (c) $f(x) = \tan^3(e^{2x})$

6. A spotlight on the ground shines on a wall 10 m away. If a woman 1.8 m tall walks from the spotlight toward the building at a speed of 2 m/s, how fast is her shadow on the building decreasing when she is 4 m from the building?

7. Let $f(x) = x\ln(x^2)$, $x > 0$. (a) Determine the intervals on which f is increasing or decreasing. (b) Determine the intervals on which f is concave up or down. (c) Find all local extreme values of f.

8. Find the point on the parabola $x + y^2 = 0$ that is closest to the point $(0, -3)$.

9. Suppose a particle travels along a straight line with velocity $v(t) = t^2 - 1$ at time t. Find its displacement and total distance traveled during the time interval $[0, 3]$.

10. Evaluate the integrals:

   (a) $\displaystyle\int \dfrac{\sin(\sqrt{x})}{\sqrt{x}}\, dx$

   (b) $\displaystyle\int_{\ln\left(\frac{\pi}{4}\right)}^{\ln\left(\frac{\pi}{3}\right)} e^x \sec^2(e^x)\, dx$