Math 662 Commutative and Homological Algebra
Homework Assignment 1
Due Friday September 20

Let R be an arbitrary ring with $1 \neq 0$.

1. Prove that chain homotopy is an equivalence relation.

2. Let B be a left R-module and let x be a nonzerodivisor of R. Show that $\operatorname{Tor}_1^R(R/xR, B) = \{b \in B \mid xb = 0\}$.

3. Let A, A', B, B' be left R-modules. Prove that
$$\begin{aligned}\operatorname{Ext}_R^n(A \oplus A', B) &\cong \operatorname{Ext}_R^n(A, B) \oplus \operatorname{Ext}_R^n(A', B), \quad \text{and} \\ \operatorname{Ext}_R^n(A, B \oplus B') &\cong \operatorname{Ext}_R^n(A, B) \oplus \operatorname{Ext}_R^n(A, B').\end{aligned}$$
(An analogous statement holds for Tor.)

4. Prove that an R-module Q is injective if and only if $\operatorname{Ext}_R^1(R/I, Q) = 0$ for all left ideals I. *(Hint: Use Baer's Criterion and a long exact sequence for* Ext.*)*

5. Let k be a field, $R = k[x]$, and I a nonzero proper ideal of R. Find $\operatorname{Tor}_n^R(R/I, R/I)$ and $\operatorname{Ext}_R^n(R/I, R/I)$ for all $n \geq 0$.

6. Let k be a field and $R = k[x]/(x^2)$. Consider k to be an R-module on which x acts as multiplication by 0. Show that $\operatorname{Tor}_n^R(k, k) \cong k$ for all $n \geq 0$.