MATH 220 Exam 1 Solutions

1. (a) For all real numbers x and y, if $x>0$ and $y>0$, then $xy>0$.
   (b) For all real numbers x and y, if $x\le 0$ or $y \le 0$, then $xy\le 0$.
   (c) There exist real numbers x and y such that $xy>0$ and $x\le 0$ or $y\le 0$.
   (d) Converse and negation

2. Let n be an odd integer, so that $n=2k+1$ for some integer k. Then $n^2+7 = (2k+1)^2+7 = 4k^2+4k+1+7 = 4k^2+4k+8 = 2(2k^2+2k+4)$, which is even since $2k^2+2k+4$ is an integer.

3. This is a biconditional statement. We will prove both:
   ($\Rightarrow$) For all integers n, if n is odd, then n^2 is odd.
   ($\Leftarrow$) For all integers n, if n^2 is odd, then n is odd.
   Proof of ($\Rightarrow$). Let n be an odd integer, so that $n=2k+1$ for some integer k. Then $n^2=(2k+1)^2 = 4k^2+4k+1 = 2(2k^2+2k)+1$, which is odd since $2k^2+2k$ is an integer.
   Proof of ($\Leftarrow$): We will prove the contrapositive, which is:
   For all integers n, if n is even, then n^2 is even.
   Proof: Let n be an even integer. Then $n=2k$ for some integer k. It follows that $n^2=(2k)^2=4k^2 = 2(2k^2)$, which is even since $2k^2$ is an integer.

4. Proof by contradiction: Assume there exist integers m and n for which $10m-6n=15$. Then $2(5m-3n)=15$. The left side of this equation is the even integer $2(5m-3n)$, while the right side is the odd integer 15. This is a contradiction, since an even integer cannot be equal to an odd integer. Therefore there do not exist such integers m, n.

5. (a) For every $\varepsilon>0$, there is a $\delta>0$ such that for all real numbers x, if $0<|x-a|<\delta$, then $|f(x)-L|<\varepsilon$.
   (b) Let $\varepsilon>0$ and $\delta=\frac{\varepsilon}{3}$. Let x be a real number for which $0<|x-2|<\delta$. Then $|(1-3x)-(-5)| = |1-3x+5| = |6-3x| = 3|2-x| = 3|x-2| < 3\cdot\delta = 3\cdot\frac{\varepsilon}{3} = \varepsilon$.
   Therefore $\lim_{x\to 2}(1-3x) = -5$.

6. F Counterexample $m=3$, $n=1$
   F Counterexample $x=0$
   T ($y=0$)
   F Counterexample $x=\sqrt{2}$, $y=-\sqrt{2}$
   F Counterexample $a=2$, $b=3$, $c=1$