Math 220 Exam 3 Practice Problems
S. Witherspoon

The following are some representative problems, from old exams, on the material for Exam 3. They are not meant to include examples of all possible problems that may be on the exam. You will also want to be prepared to work any problems similar to homework problems or those from class.

1. Let $f : \mathbb{Z} \to \mathbb{Z}$ be defined by $f(n) = \begin{cases} 2n, & \text{if } n \text{ is even} \\ n+1, & \text{if } n \text{ is odd} \end{cases}$

(a) Is f one-to-one? Justify your answer.

(b) Is f onto? Justify your answer.

2. Let $f : \mathbb{R}^2 \to \mathbb{R}^2$ and $g : \mathbb{R}^2 \to \mathbb{R}^2$ be defined by

$$f(x, y) = (-y, x) \quad \text{and} \quad g(x, y) = (x + 2, y - 1)$$

for all $(x, y) \in \mathbb{R}^2$. Find $f \circ g$ and $g \circ f$. (That is, find formulas for $(f \circ g)(x, y)$ and $(g \circ f)(x, y)$.)

3. Let $f : \mathbb{R} \to \mathbb{R}$ be defined by

$$f(x) = \begin{cases} \sqrt{x}, & \text{if } x \geq 0 \\ -x^2, & \text{if } x < 0 \end{cases}$$

Is f invertible? If so, find f^{-1}. If not, explain why not.

4. Let $f : \mathbb{Z} \to \mathbb{Z}$ be the function defined by

$$f(n) = \begin{cases} n, & \text{if } n \text{ is even} \\ 2n, & \text{if } n \text{ is odd} \end{cases}$$

(a) Find $f(\{1, 2, 3, 4\})$.

(b) Find $f^{-1}(\{1, 2, 3, 4\})$.

(c) Is f one-to-one? Justify your answer.

(d) Is f onto? Justify your answer.

5. Let A and B be sets and $f : A \to B$. Let X and Y be subsets of A.

(a) Prove that if f is *one-to-one*, then $f(X \cap Y) = f(X) \cap f(Y)$.

(b) Give an example of sets A, B, X, Y and a function f for which $f(X \cap Y) \neq f(X) \cap f(Y)$.

6. Let $f : \mathbb{R} \to \mathbb{R}$ be defined by

$$f(x) = \begin{cases} x^2, & \text{if } x \geq 0 \\ x + 1, & \text{if } x < 0 \end{cases}$$

and let $g : \mathbb{R} \to \mathbb{R}$ be defined by $g(x) = 1 - x$ for all $x \in \mathbb{R}$. Find $f \circ g$.

7. Let X, Y, Z be sets. Let $f : X \to Y$ and $g : Y \to Z$ be invertible functions. Prove that $g \circ f : X \to Z$ is invertible and that $(g \circ f)^{-1} = f^{-1} \circ g^{-1}$.