Math 365 Partial solutions to Exam 1

1. $201_{\text{five}} + 324_{\text{five}} + 1042_{\text{five}} = 2122_{\text{five}}$, $10000_{\text{five}} - 2122_{\text{five}} = 2323_{\text{five}}$

2. seven; eleven or twelve (or higher)

3. $121_{\text{four}} = 1 \cdot 4^2 + 2 \cdot 4 + 1 = 2^4 + 2^3 + 1 = 1 \cdot 2^4 + 1 \cdot 2^3 + 0 \cdot 2^2 + 0 \cdot 2 + 1 = 11001_{\text{two}}$

4. (a) Distributivity of multiplication over addition
(b) Associativity of multiplicaiton

5. In Tara's first step, she found 57-49 = 8, which is larger than 7. The remainder should be smaller than the divisor 7. Therefore she should have started with $8 \cdot 7 = 56$ instead of $7 \cdot 7 = 49$, and continued from there.

6. There are 20 terms: This is the sum of an arithmetic sequence with $a_1 = 12$ and $d = 5$. The last term is $107 = 12 + 21 \cdot 5$, using the standard form for terms in an arithemtic sequence, and so $n - 1 = 21$, that is, $n = 20$. The sum of the first and last terms is $12 + 107 = 119$. Pairing the terms, there are $20/2 = 10$ pairs, and we then find the sum of all the terms is $119 \cdot 10 = 1190$.

7. (a) 324, 972; $4 \cdot 3^{n-1}$ (geometric sequence, $r = 3$, $a_1 = 4$)
(b) 15, 21; $\dfrac{n(n+1)}{2}$ (triangular numbers, formula from class)

8. (a) **T**
(b) **F**; Counterexample: $a = 10, b = 5, c = 1$
$(10 - 5) + 1 = 6$, $10 - (5 + 1) = 4$ (many other counterexamples are possible)
(c) **F**; Counterexample: $a = 2, b = 1, c = 1$
$2 \cdot 1 + 1 = 3$, $2 \cdot (1 + 1) = 4$ (many other counterexamples are possible)
(d) **F**; Counterexample: $c = 0$ ($0 \div 0$ is undefined)