Math 367 In-class Assignment 4

Name Solutions

Notation. Let $f : A \to B$ be a function, and let T be a subset of A. Denote by $f(T)$ the subset of B consisting of all elements that can be expressed as $f(t)$ for some t in T, that is,

$$f(T) = \{f(t) \mid t \in T\}.$$

1. Let $f : \mathbb{R}^2 \to \mathbb{R}^2$, $g : \mathbb{R}^2 \to \mathbb{R}^2$, and $h : \mathbb{R}^2 \to \mathbb{R}^2$ be functions defined by

$$\begin{aligned} f((x,y)) &= (x+5, y+5), \\ g((x,y)) &= (x, -y), \\ h((x,y)) &= (-y, x). \end{aligned}$$

Let T be the triangle with vertices $(1,1)$, $(4,1)$, $(1,5)$.

(a) On graph paper, plot T, $f(T)$, $g(T)$, and $h(T)$, and label them. *(In each case, first apply the function to the vertices of T and plot the results, then connect the dots to form the new triangle.)*

(b) For each function f, g, h, briefly describe below in words what happens to the triangle T when applying the function.

(a) $f(T)$ has vertices $(1+5, 1+5)$, $(4+5, 1+5)$, $(1+5, 5+5)$, i.e. $(6,6), (9,6), (6,10)$

$g(T)$ has vertices $(1,-1), (4,-1), (1,-5)$

$h(T)$ has vertices $(-1,1), (-1,4), (-5,1)$

(b) f: shift to the right 5 units and shift up 5 units (a translation)

g: reflect over the x-axis

h: rotate $90°$ counterclockwise about the origin (other answers are possible)

2. Let $f : \mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$ be the functions defined by

$$f(x) = x + 2, \qquad g(x) = 3x$$

for all x in $\mathbb{R}$.

(a) Find the inverse functions f^{-1} and g^{-1}.

$f^{-1}(x) = x - 2$ $\qquad$ $g^{-1}(x) = \frac{1}{3}x$

To find these:

Write $y = f(x)$, i.e. $y = x - 2$

Interchange x and y:

$x = y + 2$

Solve for y:

$y = x - 2$

Write $f^{-1}(x) = x - 2$

Write $y = g(x)$, i.e. $y = 3x$

Interchange x and y:

$x = 3y$

Solve for y:

$y = \frac{1}{3}x$

Write $g^{-1}(x) = \frac{1}{3}x$

(b) Find the compositions $f \circ g$ and $g \circ f$.

$(f \circ g)(x) = f(g(x)) = f(3x) = 3x + 2$

$(g \circ f)(x) = g(f(x)) = g(x+2) = 3(x+2)$ or $3x + 6$

(c) Find the inverses of the compositions $(f \circ g)^{-1}$ and $(g \circ f)^{-1}$.

Solution 1. $(f \circ g)^{-1}(x) = (g^{-1} \circ f^{-1})(x) = g^{-1}(f^{-1}(x)) = g^{-1}(x-2) = \frac{1}{3}(x-2)$

$(g \circ f)^{-1}(x) = (f^{-1} \circ g^{-1})(x) = f^{-1}(g^{-1}(x)) = f^{-1}(\frac{1}{3}x) = \frac{1}{3}x - 2$

Solution 2. Use the method of part (a), e.g. to find $(f \circ g)^{-1}$,

write $y = (f \circ g)(x)$, i.e. $y = 3x + 2$ (from part (b))

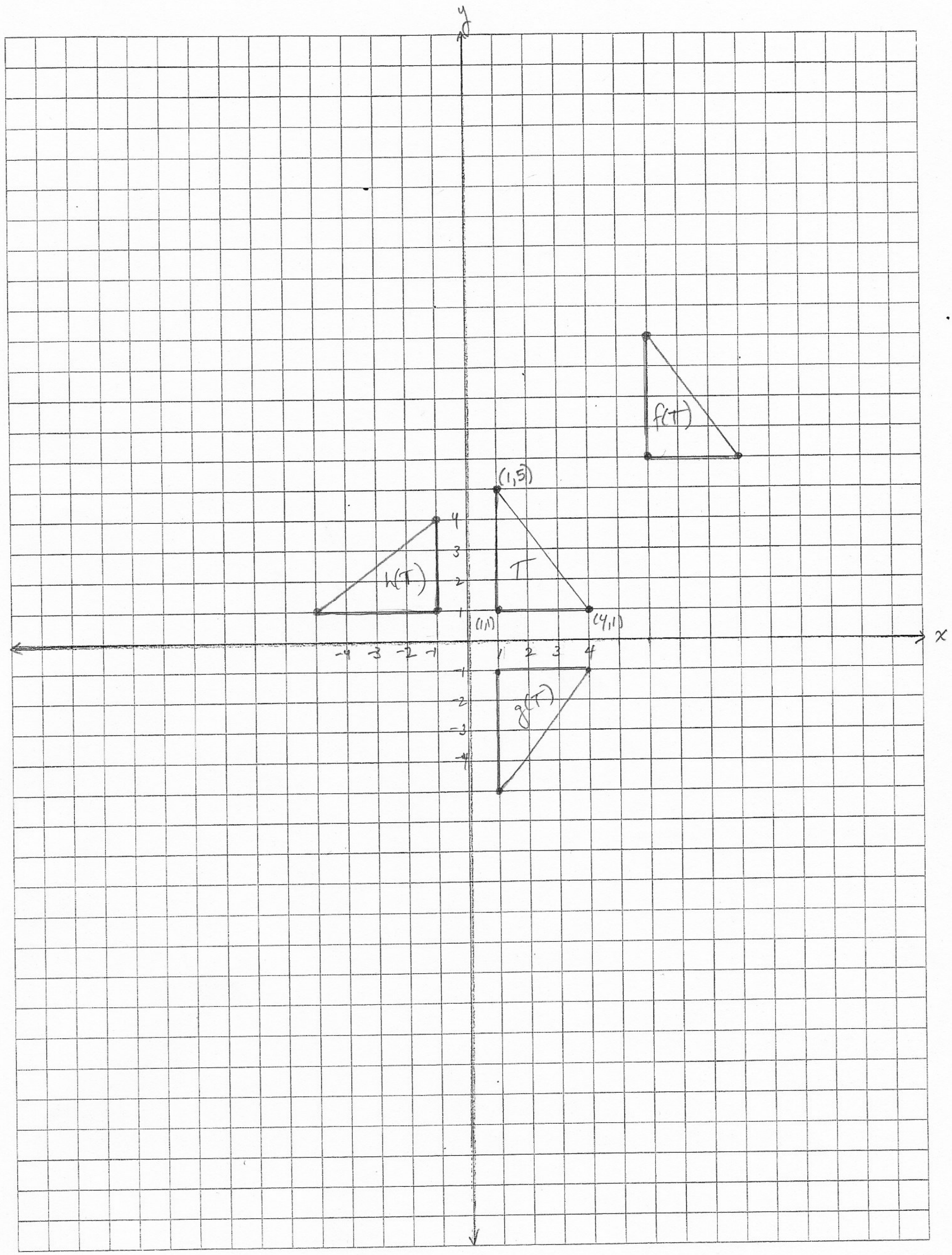
Interchange x and y: $x = 3y + 2$

Solve for y: $3y = x - 2$

$y = \frac{1}{3}(x-2)$

Write $(f \circ g)^{-1}(x) = \frac{1}{3}(x-2)$.

Finding $(g \circ f)^{-1}(x)$ is similar.



y
x
(1,5)
(1,1)
(4,1)
T
f(T)
g(T)
h(T)