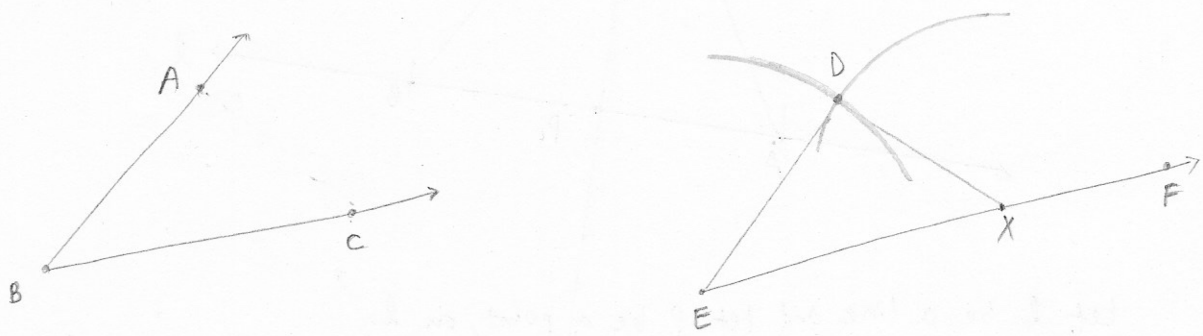
Math 367 In-class Assignment 6

Name Solutions

The following is a modified version of a theorem from the book:

Theorem 31. Given an angle $\angle ABC$ and a ray $\overrightarrow{EF}$, there is a ray $\overrightarrow{ED}$ so that $\angle ABC \cong \angle DEF$.

1. Prove Theorem 31 in two steps, (a) and (b), as follows:
(a) First explain that there is a triangle congruent to $\triangle ABC$, one of whose sides is on the ray $\overrightarrow{EF}$. *(Hint: First use Theorem 29 to explain that there is a point X on $\overrightarrow{EF}$ such that $BC \cong EX$. Then explain that there is a point D for which $\triangle ABC \cong \triangle DEX$ by recalling how to construct such a triangle $\triangle DEX$ using straightedge and compass.)*



By Theorem 29, applied to the segment BC and ray $\overrightarrow{EF}$, there is a point X on $\overrightarrow{EF}$ such that $BC \cong EX$.

Let D be the intersection point of two circles:
① the circle centered at X with radius $\ell(CA)$
② the circle centered at E with radius $\ell(BA)$

Then: $BC \cong EX$, $BA \cong ED$, $CA \cong XD$.

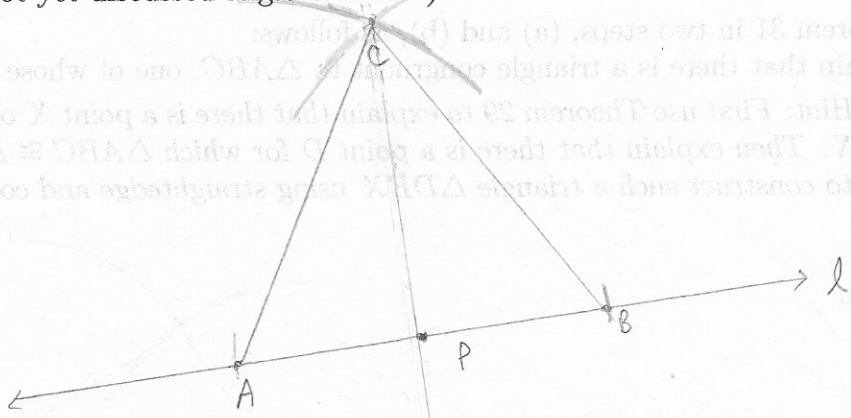
By the SSS Axiom, $\triangle ABC \cong \triangle DEX$.

(b) Next explain how to apply CPCFC (or CPCTC) to the triangles from part (a) to prove Theorem 31.

Since $\triangle ABC \cong \triangle DEX$, by CPCFC, $\angle ABC \cong \angle DEF$.

Theorem 34. There is a line perpendicular to any given line through a given point on the line.

2. Prove Theorem 34. *(Hint: Think about a construction, using straightedge and compass, of a perpendicular line. Then write the proof by explaining what you did, and explaining why the line is perpendicular by using triangle congruence axiom(s). Recall that a right angle was defined to be an angle that is congruent to one of its supplements. We have not yet discussed angle measure.)*



Let l be a line and let P be a point on l.

Let A be another point on l.

Let B be a point on l:

(B is the other point of intersection of l and the circle with center P and radius $\angle(AP)$, so $AP \cong BP$.)

Let C, D be intersection points of two circles:

① the circle with center B and radius $\angle(AB)$ (or a little less)

② the circle with center A and radius $\angle(AB)$ (or a little less)

We will show that $\overleftrightarrow{CD}$ is perpendicular to l.

We have already seen that $AP \cong BP$.

By construction, $AC \cong BC$. Since $CP \cong CP$ as well, the SSS Axiom implies that $\triangle ACP \cong \triangle BCP$.

By CPCFC, $\angle APC \cong \angle BPC$.

Since $\angle APC$ and $\angle BPC$ are supplementary, and also congruent, they are right angles. Therefore $\overleftrightarrow{PC}$ is perpendicular to l.