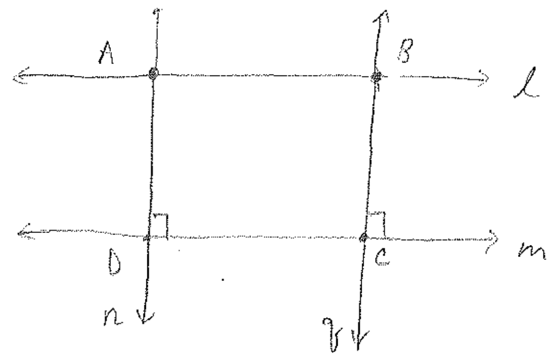
Math 367 In-class Assignment 8

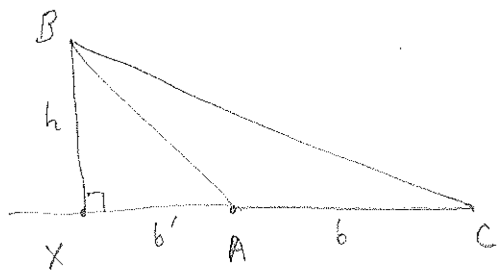
Name Solutions

Prove **Theorem 63**: *Assume that ℓ and m are two parallel lines. Then all points on m are the same distance from ℓ.*



Proof Let A and B be any two distinct points on ℓ. By Theorem 33, there is a line n perpendicular to m through A, and a line g perpendicular to m through B. By Theorem 60, since ℓ and m are parallel, the alternate interior angles are congruent, and so all angles formed by these intersecting lines are right angles. Let C and D be the points at which g and n intersect m, respectively. By definition, $\square ABCD$ is a rectangle. By Corollary 62, $AD \cong BC$. So A and B are the same distance from ℓ. $\square$

Do **Problem 70**: Let $\triangle ABC$ be a triangle and let X be the point on $\overleftrightarrow{AC}$ such that $\overleftrightarrow{BX}$ is perpendicular to $\overleftrightarrow{AC}$. Let $h = \mathcal{L}(BX)$ and $b = \mathcal{L}(AC)$. Prove that if A is between X and C, then $\mathcal{A}(\triangle ABC) = \frac{1}{2}bh$.



Proof let $b' = \mathcal{L}(XA)$.
By the solution to Problem 68, the areas of the two right triangles formed are:

$$\mathcal{A}(\triangle BXC) = \frac{1}{2}(b'+b)h, \quad \mathcal{A}(\triangle BXA) = \frac{1}{2}b'h.$$

By Axiom 6(iii), $\mathcal{A}(\triangle BXC) = \mathcal{A}(\triangle BXA) + \mathcal{A}(\triangle BAC)$, so

$$\frac{1}{2}(b'+b)h = \frac{1}{2}b'h + \mathcal{A}(\triangle BAC).$$

Solving for $\mathcal{A}(\triangle BAC)$, we have:

$$\begin{aligned}\mathcal{A}(\triangle BAC) &= \frac{1}{2}(b'+b)h - \frac{1}{2}b'h \\ &= \frac{1}{2}b'h + \frac{1}{2}bh - \frac{1}{2}b'h \\ &= \frac{1}{2}bh. \quad \square\end{aligned}$$