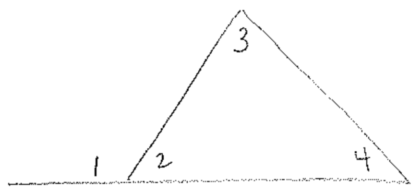
Math 367 In-class Assignment 9

Name Solutions

Prove the **Strong Exterior Angle Theorem 90**: *The measure of an exterior angle of a triangle is equal to the sum of the measures of the two opposite interior angles.*



By Theorem 87, $\mathcal{D}(\angle 2) + \mathcal{D}(\angle 3) + \mathcal{D}(\angle 4) = 180°$.

By Theorem 86, $\mathcal{D}(\angle 1) + \mathcal{D}(\angle 2) = 180°$.

So:

$$\mathcal{D}(\angle 2) + \mathcal{D}(\angle 3) + \mathcal{D}(\angle 4) = \mathcal{D}(\angle 1) + \mathcal{D}(\angle 2),$$

and subtracting $\mathcal{D}(\angle 2)$ from both sides, we find that

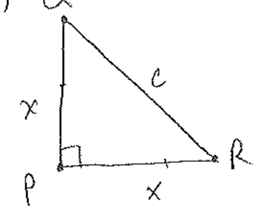
$$\mathcal{D}(\angle 3) + \mathcal{D}(\angle 4) = \mathcal{D}(\angle 1).$$

Do **Problem 101**: An **isosceles right triangle** is a right triangle with congruent legs.

(a) Let x be the length of the legs of an isosceles right triangle. Show that the hypotenuse has length $x\sqrt{2}$ and that the base angles both have measure 45°.

(b) Find the area of a square with diagonal of length d.

(a)



By the Pythagorean Theorem,

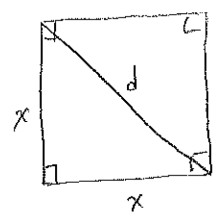
$$x^2 + x^2 = c^2$$
$$2x^2 = c^2$$
$$\sqrt{2x^2} = c$$
$$x\sqrt{2} = c$$

By Theorem 36, the base angles are congruent: $\angle PQR \cong \angle PRQ$.
By Theorem 87, $90° + \mathcal{D}(\angle PQR) + \mathcal{D}(\angle PRQ) = 180°$, so

$$\mathcal{D}(\angle PQR) + \mathcal{D}(\angle PRQ) = 90°.$$

Since $\mathcal{D}(\angle PQR) = \mathcal{D}(\angle PRQ)$, both must be 45°.

(b)



From part (a), $d = c = x\sqrt{2}$, so $x = \frac{d}{\sqrt{2}}$.

By Theorem 73, the area is $x \cdot x = x^2 = \left(\frac{d}{\sqrt{2}}\right)^2 = \frac{d^2}{2}$.