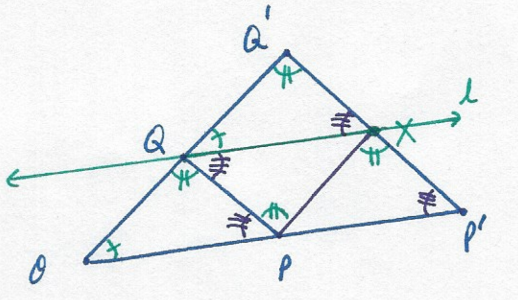
Theorem 112 If a dilation has a scaling factor 2, then the length of every segment PQ is multiplied by 2, that is, $\ell(P'Q') = 2\ell(PQ)$.

Proof in the case that the center O of the dilation is not on $\overleftrightarrow{PQ}$:



Let l be the line through Q and parallel to $\overleftrightarrow{OP}$ (l exits by Cor 42).

Let X be the intersection of l and $\overleftrightarrow{P'Q'}$.

By Cor 61, $\angle QOP \cong \angle Q'QX$.

By Thm 110, $\overleftrightarrow{Q'P'}$ is parallel to $\overleftrightarrow{QP}$.

By Cor 61, $\angle OQP \cong \angle QQ'P'$.

By construction, $OQ \cong QQ'$ (since the scaling factor is 2), so by the ASA Thm, $\triangle OQP \cong \triangle QQ'X$. By CPCFC, $QX \cong OP$, $QP \cong Q'X$.

By Thm 60, $\angle Q'XQ \cong \angle XQP$. By the SAS Axiom, $\triangle QXQ' \cong \triangle XQP$. By CPCFC, $PX \cong OQ$. By construction, $PP' \cong OP$. By Thm 60, $\angle QPX \cong \angle PXP'$. By the AAS Thm, $\triangle XPP' \cong \triangle QOP$. So by CPCFC, $XP' \cong QP$. Combining all of the above, we see that

$$\begin{aligned}\ell(Q'P') &= \ell(Q'X) + \ell(XP') \\ &= \ell(PQ) + \ell(PQ) \\ &= 2\ell(PQ). \qquad \square\end{aligned}$$