Math 367 Exam 1
SOLUTIONS

Yellow solutions

1. (a) For all integers m and n, if m and n are even, then $m+n$ is even.
(b) For all integers m and n, if m is odd or n is odd, then $m+n$ is odd.
(c) There exist integers m and n such that $m+n$ is even and m is odd or n is odd.
(d) The converse (a) and the negation (c) are true; the others are false. (Justification was not required, but here it is: The original statement is false, and $m=3$, $n=5$ is a counterexample since $3+5=8$ is even and it is not true that m and n are even. The converse is true, and may be proven using our standard techniques. The contrapositive is false for the same reason as given for the original statement. The negation is true since the original statement is false, and the counterexample given above also shows this.)

2. Let m and n be odd integers. Then $m=2k+1$ and $n=2p+1$ for some integers k and p. So $m-n=(2k+1)-(2p+1)=2k+1-2p-1=2k-2p=2(k-p)$, which is even.

3. (a) $f^{-1}(x)=\dfrac{x-3}{2}$, $g^{-1}(x)=\dfrac{x}{5}$
(b) $(f\circ g)(x)=f(g(x))=f(5x)=2(5x)+3=10x+3$
$(g\circ f)(x)=g(f(x))=g(2x+3)=5(2x+3)=10x+15$

4. F; counterexample: $m=3$ for which $m^2+m=9+3=12$
F; counterexample: $m=3$, $n=2$, for which $m+n=5$
F; counterexample: $n=-3$

5. No. There are many possible explanations, such as either of the following:
(i) Writing $y=x^2+2$ and interchanging x and y, we want to solve $x=y^2+2$ for y. We obtain $y=\pm\sqrt{x-2}$, that is, for some values of x, there are two y-values. An inverse function would have to give exactly one y-value, so an inverse function cannot exist. (Put another way, if f were invertible, then e.g. $f^{-1}(3)$ would have to equal both 1 and -1 since $f(1)=3$ and $f(-1)=3$, but then f^{-1} would not be a function.)
(ii) f cannot be invertible, since if it were, then $f^{-1}(-1)$ would have to be equal to a number that f takes to -1, however, there are no negative numbers as function values, and so there is no such function.

6. $(3,-1)$, $(4,-1)$, $(4,9)$

7. F, T, F (No justification was required, but here it is: By its definition, $g\circ f$ is a function from A to C, so its domain is A. If f is invertible and g is invertible, we found in class that $g\circ f$ is invertible with inverse $f^{-1}\circ g^{-1}$.)

White solutions

1. (a) There exist integers m and n such that $m+n$ is even and m is odd or n is odd.
(b) For all integers m and n, if m and n are even, then $m+n$ is even.
(c) For all integers m and n, if m is odd or n is odd, then $m+n$ is odd.
(d) The negation (a) and the converse (b) are true; the others are false. (Justification was not required, but here it is: The original statement is false, and $m=3$, $n=5$ is a counterexample since $3+5=8$ is even and it is not true that m and n are even. The negation is true since the original statement is false, and the counterexample just above also shows this. The converse is true, and may be proven using our standard techniques. The contrapositive is false for the same reason as given for the original statement.)

2. Let m and n be odd integers. Then $m=2k+1$ and $n=2p+1$ for some integers k and p. So $m-n=(2k+1)-(2p+1)=2k+1-2p-1=2k-2p=2(k-p)$, which is even.

3. (a) $f^{-1}(x)=\dfrac{x-2}{5}$, $g^{-1}(x)=\dfrac{x}{3}$
(b) $(f\circ g)(x)=f(g(x))=f(3x)=5(3x)+2=15x+2$
$(g\circ f)(x)=g(f(x))=g(5x+2)=3(5x+2)=15x+6$

4. F; counterexample: $m=-2$
F; counterexample: $n=3$ for which $n^2+n=9+3=12$
F; counterexample: $m=2$, $n=3$, for which $m+n=5$

5. No. There are many possible explanations, such as either of the following:
(i) Writing $y=x^2+3$ and interchanging x and y, we want to solve $x=y^2+3$ for y. We obtain $y=\pm\sqrt{x-3}$, that is, for some values of x, there are two y-values. An inverse function would have to give exactly one y-value, so an inverse function cannot exist. (Put another way, if f were invertible, then e.g. $f^{-1}(4)$ would have to equal both 1 and -1 since $f(1)=4$ and $f(-1)=4$, but then f^{-1} would not be a function.)
(ii) f cannot be invertible, since if it were, then $f^{-1}(-1)$ would have to be equal to a number that f takes to -1, however, there are no negative numbers as function values, and so there is no such function.

6. $(2,-1)$, $(3,-1)$, $(3,11)$

7. T, F, F (No justification was required, but here it is: If f is invertible and g is invertible, we found in class that $g\circ f$ is invertible with inverse $f^{-1}\circ g^{-1}$. By its definition, $g\circ f$ is a function from A to C, so its domain is A.)