**Math 367 Homework Assignment 1**
*SOLUTIONS*

1. (a) There is a real number $x$ such that $x^3 \leq 0$. *(There are many other possible such statements, for example, "There exists a real number $x$ for which $x^3$ is not greater than 0.")*
   (b) All squares are rhombuses.
   (c) There are integers $m$ and $n$ such that $mn$ is even and $m$ is not even. *(There are many other possible such statements, for example, "not even" could be replaced by "odd".)*
   (d) Sam is not a fourth grader or Rachel is not a fourth grader.
   (e) Mandy was not born in Oklahoma and Jack was not born in Oklahoma.

2. (a) FALSE. Counterexample: $x = -2$ *(Any negative number, or 0, is a counterexample.)*
   (b) FALSE. Counterexample: $m = 3$, $n = 2$ *(There are many other counterexamples.)*
   (c) TRUE. Proof: Let $m$ and $n$ be even integers. Then $m = 2k$ and $n = 2p$ for some integers $k$ and $p$. Their difference is

$$m - n = 2k - 2p = 2(k - p),$$

which is even.
   (d) FALSE. Counterexample: Let $m = 4$, $n = 6$. Then $m + n = 10$, which is even, and $m$ and $n$ are not odd.

3. Let $n$, $n+1$, $n+2$ be consecutive integers. Then

$$n + (n+1) + (n+2) = 3n + 3 = 3(n+1),$$

which is divisible by 3.

4. (a) (i) TRUE.
(ii) If $x^2 = 4$, then $x = 2$. FALSE. Counterexample: $x = -2$
   (b) (i) FALSE. Counterexample: $x = -3$
(ii) If $x = 3$, then $|x| = 3$. TRUE.
   (c) (i) FALSE. Counterexample: $x = -2$
(ii) If $x^2 < 1$, then $x < 1$. TRUE.
   (d) (i) TRUE. (ii) If $x^3 < 1$, then $x < 1$. TRUE.

5. The contrapositive is: For all integers $m$ and $n$, if $m$ is even or $n$ is even, then $mn$ is even.
   Proof: Let $m$ and $n$ be integers. Suppose first that $m$ is even. Then $m = 2k$ for some integer $k$. It follows that

$$mn = (2k)n = 2(kn),$$

which is even. In case $m$ is odd and $n$ is even, a similar calculation shows that $mn$ is even.

6. (1) For all integers $n$, if $n$ is even, then $n^3$ is even.
Proof: Let $n$ be an even integer, so that $n = 2k$ for some integer $k$. Then $n^3 = (2k)^3 = 8k^3 = 2(4k^3)$, which is even.

   (2) For all integers $n$, if $n^3$ is even, then $n$ is even.
The contrapositive of this statement is
For all integers $n$, if $n$ is odd, then $n^3$ is odd.
Proof: Let $n$ be an odd integer. Then $n = 2k+1$ for some integer $k$. It follows that

$$\begin{aligned} n^3 = (2k+1)^3 &= (2k+1)(2k+1)^2 \\ &= (2k+1)(4k^2+4k+1) \\ &= 8k^3 + 8k^2 + 2k + 4k^2 + 4k + 1 \\ &= 8k^3 + 12k^2 + 6k + 1 \\ &= 2(4k^3 + 6k^2 + 3k) + 1, \end{aligned}$$

which is odd.