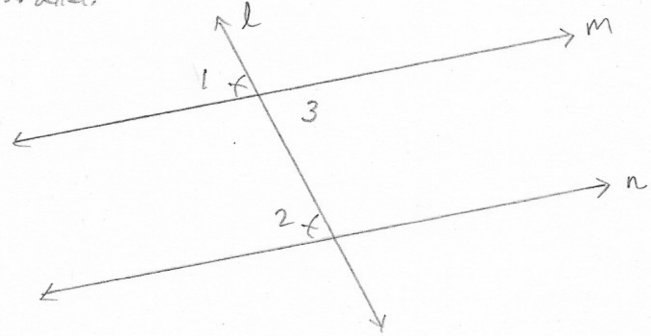
MATH 367 - Homework Assignment 5 - Solutions

Corollary 41. If two lines have a transversal which forms corresponding angles that are congruent, then the two lines are parallel.

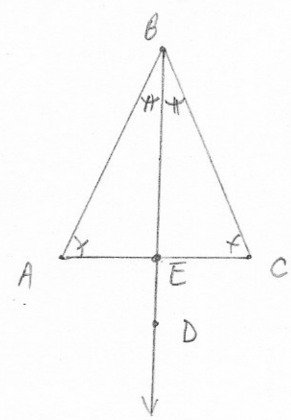


Proof: Let l be a transversal of two lines m and n for which corresponding angles are congruent. Label two corresponding congruent angles $\angle 1$ and $\angle 2$ as shown.

By Corollary 38, vertical angles are congruent. Label as $\angle 3$ the angle which together with $\angle 1$ forms a pair of vertical angles. So $\angle 3 \cong \angle 1$. It follows that $\angle 3 \cong \angle 2$ (since $\angle 1 \cong \angle 2$). Now $\angle 3$ and $\angle 2$ are alternate interior angles. By Theorem 40, m and n are parallel. $\square$

Thm 46 If two angles of a triangle are congruent, then the opposite sides are congruent.

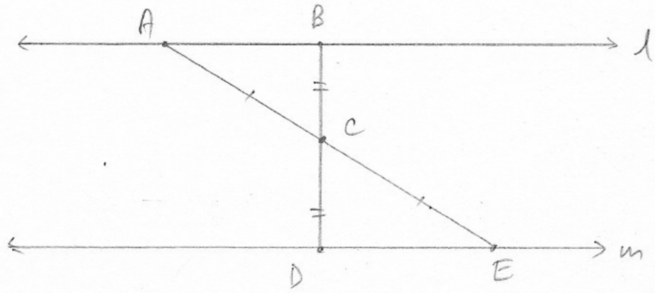
Proof: (Note that this is the converse of Theorem 36.) Let $\triangle ABC$ be a triangle having two angles congruent, say $\angle BAC \cong \angle BCA$. By Theorem 32, $\angle ABC$ has a bisector $\overrightarrow{BD}$ for some point D. So $\angle ABD \cong \angle CBD$. Let E be the intersection point of $\overrightarrow{BD}$ and AC. Since $BE \cong BE$, by Thm 45 (AAS), $\triangle ABE \cong \triangle CBE$.



By CPCFC, $AB \cong CB$. $\square$

Problem 56. By construction, $AC \cong CE$ and $BC \cong CD$. Since $\angle ACB$ and $\angle ECD$ are vertical angles, by Cor. 38, they are congruent. By Axiom 3 (SAS),

$$\triangle ABC \cong \triangle EDC.$$



By CPCFC, $\angle BAC \cong \angle DEC$. These two angles are alternate interior angles with respect to the transversal $\overleftrightarrow{AE}$ of lines l, m (which correspond to the first and second floor frames). By Thm 40, l and m are parallel.

Problem 57.

(iv) The vertical angles are congruent. By the AAS Theorem, the two triangles are congruent. So $x = 32$, $y = 25$, by CPCFC.

(v) Let M be the intersection of IK and JL. Then $\triangle LMK$ is isosceles, so by Thm 36, $\angle MLK \cong \angle MKL$. Now consider the two right triangles, $\triangle ILK$ and $\triangle JKL$. Since $KL \cong KL$, by the ASA Thm (or by Cor 52 (LA)), these two right triangles are congruent. So $x = 25$, by CPCFC.

(vii) By Thm 48 (HL), $\triangle PSQ \cong \triangle RQS$. By CPCFC, $x = 14$.

(viii) By Thm 44 (ASA), $\triangle ACD \cong \triangle CAB$. So by CFCFC, $x = 30$, $y = 20$.