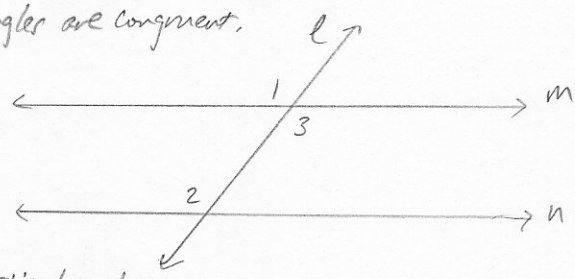
MATH 367 - Homework Assignment 6 - Solutions

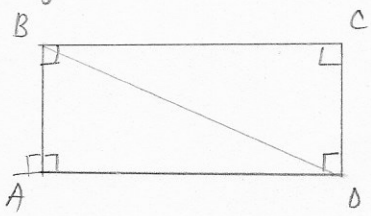
Corollary 61 Assume a transversal line intersects two lines. Then the two lines are parallel if and only if the corresponding angles are congruent.



Proof Let ℓ be a transversal of lines m, n.
($\Rightarrow$) Assume m and n are parallel.
By Thm 60, alternate interior angles are congruent: $\angle 2 \cong \angle 3$.
By Cor 38, $\angle 1 \cong \angle 3$ since they are vertical angles.
Therefore $\angle 1 \cong \angle 2$, that is, corresponding angles are congruent.

($\Leftarrow$) Assume corresponding angles are congruent: $\angle 1 \cong \angle 2$.
By Cor 41, lines m and n are parallel. □

Corollary 62 Opposite sides of a rectangle are congruent.



Proof Let $\square ABCD$ be a rectangle.
By definition of a rectangle, the four sides form four right angles.
By definition, a right angle is congruent to a supplement.
Consider lines $\overleftrightarrow{BC}$ and $\overleftrightarrow{AD}$, with transversal $\overleftrightarrow{AB}$: Alternate interior angles are congruent (they are right angles), so by Thm 60, $\overleftrightarrow{BC}$ and $\overleftrightarrow{AD}$ are parallel.
Now consider $\overleftrightarrow{BD}$ to be a transversal of $\overleftrightarrow{BC}$ and $\overleftrightarrow{AD}$. By Thm 60, alternate interior angles are congruent, so $\angle CBD \cong \angle ADB$.
By similar arguments to the above, $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ are parallel, and $\angle ABD \cong \angle CDB$.
Clearly $BD \cong BD$, and so by Thm 44 (ASA), $\triangle ABD \cong \triangle CDB$.
By CPCFC, $AB \cong CD$ and $BC \cong AD$. □

Problem 76 Answers will vary somewhat. I counted 169 squares, so $A(X) = 169$ square units (or 16,900 square miles, since each unit square has area $10 \times 10 = 100$ square miles)

Problem 77 $A(Y) = 289$ square units (or 28,900 square miles)

Problem 78 (i) $169 \leq A(WV) \leq 289$ (or 16,900 sq. miles $\leq A(WV) \leq$ 28,900 sq. mi)
(ii) $\frac{1}{2}(16,900 + 28,900) = 22,900$ square miles
(iii) 234 unit squares, or 23,400 square miles
(iv) Encyclopaedia Britannica (online): 24,230 square miles
Estimate (iii) is closer.

Problem 79 Choose a smaller unit, e.g. squares whose side lengths are 1 mile instead of 10 miles.