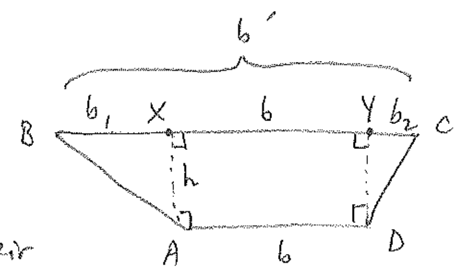
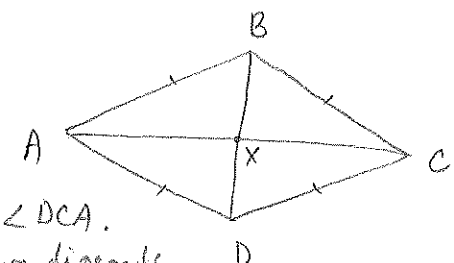
MATH 367 HW 7 SOLUTIONS



Theorem 74 Proof:

Assume $b' > b$ as in the picture.
Consider line segments perpendicular to $\overline{AD}$ and through A and D, and their intersection points X and Y with $\overleftrightarrow{BC}$.
Since $\overleftrightarrow{BC}$ is parallel to $\overleftrightarrow{AD}$ by definition of trapezoid, by Theorem 60, $\square AXYD$ is a rectangle, with base b and height h. The triangles $\triangle AXB$ and $\triangle CYD$ have bases b_1 and b_2, respectively, and height h. So

$$A(\square ABCD) = A(\triangle AXB) + A(\triangle CYD) + A(\square AXYD)$$
$$= \frac{1}{2} b_1 h + \frac{1}{2} b_2 h + bh$$
$$= \frac{b_1 h + b_2 h + 2bh}{2}$$
$$= \frac{(b_1 + b_2 + b + b)h}{2}$$
$$= \frac{(b + b')h}{2}. \quad \square$$



Theorem 75 Proof:

(i) First note that by the SSS Axiom, $\triangle ABC \cong \triangle ADC$.
By CPCFC, $\angle BAC \cong \angle DAC$ and $\angle BCA \cong \angle DCA$.
Let X be the intersection point of the two diagonals.
By the SAS Axiom, $\triangle ABX \cong \triangle ADX$. Similarly, $\triangle BCX \cong \triangle DCX$.
Similar arguments show that $\triangle ABD \cong \triangle CBD$ and $\triangle ABX \cong \triangle CBX$.
Thus all four of these smaller triangles are congruent.

(ii) Note that each of the four small triangles has base $\frac{d_1}{2}$ and height $\frac{d_2}{2}$ (or vice versa, if viewed differently). So

$$A(\square ABCD) = 4A(\triangle ABX)$$
$$= 4 \cdot \frac{1}{2}\left(\frac{d_1}{2}\right)\left(\frac{d_2}{2}\right)$$
$$= \frac{d_1 d_2}{2}. \quad \square$$

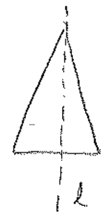
1. (Answers may vary, depending on the font used!)

(a) A, H, I, M, O, T, U, V, X, Y

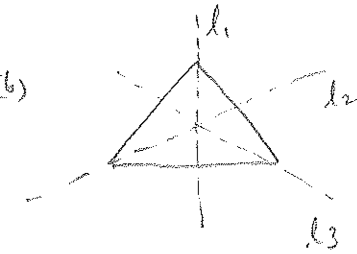
(b) B, C, D, E, H, I, K, O, X

(c) H, I, O, X

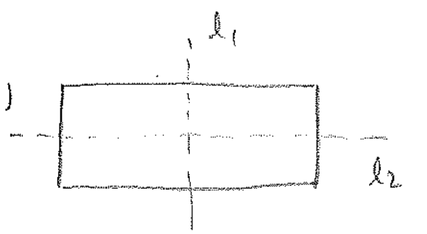
2. (a)



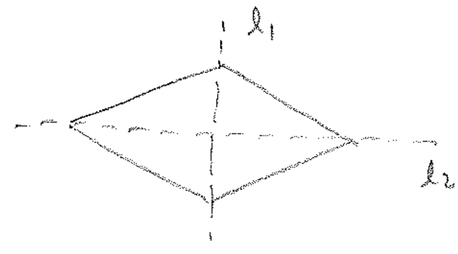
(b)



(c)



(d)



(e)