MATH 367 HW8 SOLUTIONS

81. (i) Since $\overrightarrow{XD}$ bisects $\angle CXF$, there is a congruence of angles: $\angle CXD \cong \angle FXD$. Since $\angle AXC$ is a right angle, so is $\angle CXF$, and so $\mathcal{D}(\angle CXF) = 90°$. So

$$\mathcal{D}(\angle CXD) + \mathcal{D}(\angle FXD) = 90°$$
$$2\mathcal{D}(\angle FXD) = 90°$$
$$\mathcal{D}(\angle FXD) = 45°$$

(ii) Since $\overrightarrow{XE}$ bisects $\angle FXD$ and $\mathcal{D}(\angle FXD) = 45°$, it follows that $\mathcal{D}(\angle FXE) = 22.5°$.

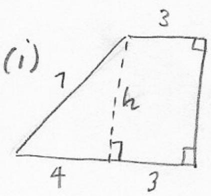
(iii) Since $\overrightarrow{XB}$ bisects $\angle AXC$, which is a right angle, it follows that $\mathcal{D}(\angle CXB) = 45°$. So $\mathcal{D}(\angle FXB) = \mathcal{D}(\angle CXB) + \mathcal{D}(\angle CXF) = 45° + 90° = 135°$.

94. $c^2 = \sqrt{12^2 + 7^2} = \sqrt{144+49} = \sqrt{193} \approx 13.89$

95. (Answers will vary a little.) $\sqrt{9^2 + 8^2} \approx 12$
120 miles

96. $\sqrt{11^2 + 26^2} \approx 28$
280 miles

102. (i)

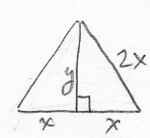


$4^2 + h^2 = 7^2 \rightsquigarrow h \approx 5.7$
So the area is approx. $\frac{1}{2}(4)(5.7) + 3(5.7) = 28.5$

(ii) Since $3^2 + 4^2 = 5^2$, by Thm 100, the triangle is a right triangle, and we can take the base and height to be 3 and 4. So the area is $\frac{1}{2}(3)(4) = 6$.

(iii) Similarly to (ii), the area is $\frac{1}{2}(12)(5) = 30$.

103. (i) One way to do this is to start with an equilateral triangle and bisect one of the angles. This produces two 30-60-90 triangles.



(ii) The two triangles from (i) are congruent by SAS. Letting x be the length of a short leg, the hypotenuse has length $2x$ by construction. Let y be the length of the other leg. Then $y^2 = (2x)^2 - x^2$ by the Pythagorean Theorem, so $y = \sqrt{3x^2}$, i.e. $y = x\sqrt{3}$.

(iii) Set $s = 2x$ from parts (i) and (ii), so $x = \frac{s}{2}$, and the area is

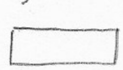
$$2 \cdot \frac{1}{2}xy = 2 \cdot \frac{1}{2} \cdot x \cdot x\sqrt{3} = \frac{s}{2} \cdot \frac{s}{2} \cdot \sqrt{3} = \frac{\sqrt{3}}{4}s^2.$$

1. (a) H, I, N, O, S, X, Z (b) O (if written as a circle, otherwise, none)

2. (a)

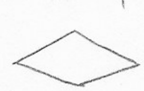
120°, 240°, (360°)

(b)



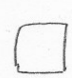
180° (360°)

(c)



180° (360°)

(d)



90°, 180°, 270°, (360°)