ERRATUM TO "GERSTENHABER BRACKETS ON HOCHSCHILD COHOMOLOGY OF GENERAL TWISTED TENSOR PRODUCTS"

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Abstract. In our paper [1], several maps in Section 4 were incorrect. We correct them here. This oversight does not affect any of the results in the paper.

The techniques developed in [1, Section 3] are showcased for the Jordan plane in Section 4. It is stated shortly after display (4.1) that for $(i,j) \in \{(1,1),(1,0),(0,1)\}$:

$$P_i \otimes Q_j \coloneqq A \otimes A \otimes B \otimes B \xrightarrow{1\otimes\tau^{-1}\otimes 1} (A \otimes_\tau B) \otimes (A \otimes_\tau B)^{op}$$

is an isomorphism of $(A \otimes_\tau B)^e$-modules. This statement is incorrect because in these cases $1 \otimes \tau^{-1} \otimes 1$ is not a morphism of $(A \otimes_\tau B)^e$-modules.

The sole purpose of the statement above is to prove that $P_i \otimes Q_j$ is a free $(A \otimes_\tau B)^e$-module of rank one. We now establish this by describing a correct isomorphism

$$\varphi : (A \otimes_\tau B) \otimes (A \otimes_\tau B)^{op} \to P_i \otimes Q_j.$$

The free $(A \otimes_\tau B)^e$-module $(A \otimes_\tau B) \otimes (A \otimes_\tau B)^{op}$ is of rank one with free basis element $(1 \otimes 1) \otimes (1 \otimes 1)$. Define $\varphi((1 \otimes 1) \otimes (1 \otimes 1)) \coloneqq e_i \otimes e'_j$ in $P_i \otimes Q_j$ and let φ be the $(A \otimes_\tau B)^e$-module homomorphism generated by this choice. It satisfies

$$\varphi((x^a \otimes y^b) \otimes (x^{a'} \otimes y^{b'})) \coloneqq (x^a \otimes y^b)(e_i \otimes e'_j)(x^{a'} \otimes y^{b'}) = (x^a \otimes y^b)(e_i x^{a'} \otimes e'_j y^{b'}).$$

This equals $x^a e_i x^{a'} \otimes y^b e'_j y^{b'} + S$, where S is a sum of terms whose degree in y is less than $b + b'$ and whose total degree is still $a + a' + b + b'$. (Recall that the $(A \otimes_\tau B)^e$-actions on $P_i \otimes Q_j$ are given by the maps $\tau_{0,A}$, $\tau_{1,A}$, $\tau_{B,0}$, and $\tau_{B,1}$ of [1, Section 4].) By induction on the degree in y, the top term $x^a e_i x^{a'} \otimes y^b e'_j y^{b'}$ of this expression is also in the image of the map φ. Therefore φ is surjective. By construction, φ can be expressed via an upper triangular matrix in each total polynomial degree, so it is also injective.

Replacing the incorrect map $1 \otimes \tau^{-1} \otimes 1$ with φ^{-1} fixes the argument in [1].

References

[1] T. Karadağ, D. McPhate, P. S. Ocal, T. Oke, and S. Witherspoon, *Gerstenhaber brackets on Hochschild cohomology of general twisted tensor products*, J. Pure Appl. Algebra 225 (2021), no. 6, 14 pp.

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